Wage posting and multidimensional skills mismatch

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Abstract

This paper gives a new answer to an old question in labor economics, *"Who matches with whom?"*, by introducing a setting where firms and workers are different in many dimensions and we allow workers to be over and under qualified for the jobs they end up occupying. I present a random search model with two sided multidimensional heterogeneity in which firms choose and post a wage with commitment i.e. maintaining the posted wage, independent of the productivity of the new worker. Posted wages determine the set of acceptable jobs for each worker and a unique *applicants pool* for each firm. The composition of these sets varies in size and composition across workers and firms. The optimal posted wage level takes into consideration the requirements of each firm and the characteristics of the applicants pool. In equilibrium, sorting is assortative but mismatches can occur across all skills dimensions. Using French data on workers observed skills and matches, I estimate the structural parameters associated with the model for France. I find that the disutility of non cognitive skills is higher when mismatched, while employers value more highly good matches on cognitive skills. I also find that the number of dimensions plays an important role, since it is another source for frictions.

Keywords: Wage posting, skills mismatch, search and matching

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Introduction

Policy makers are often concerned about the possible losses of productivity for firms that do not find a suitable workforce, since this slows firms' growth and competitiveness. Contemporaneously, there have been increasing concerns about workers' well-being and dissatisfaction due to over-qualification and under-qualification. These two phenomena are just two sides of the same coin and are the consequences of mismatch in the labor market. An old question then re-arises: "*Who matches with whom?*". This paper provides a new answer to this old question by providing a microeconomic explanation based on worker and firm behaviour when skills are multidimensional, which allow us to understand how workers and firms sort in an environment where mismatches are the norm.

This paper makes the following contributions. First, the paper contributes to the (limited) literature on multidimensional search literature (Lindenlaub and Postel-Vinay, 2016; Lise and Postel-Vinay, 2015; Tan, 2017; Lazear, 2009), proposing a microfoundation for understanding how matching and sorting occurs in the case of multiple and heterogeneous skills. Moreover, this paper contributes to the literature of multidimensional skills, providing a theoretical evidence for how wages depend on multidimensional skills and requirements (Deming, 2017; Deming and Kahn, 2018; Speer, 2017). When we take the model to the data, we also find that workers suffer greater disutility for mismatches in the non cognitive skill dimension. Firms, on the other hand, value better matches on the cognitive skill dimension more highly. Lastly, the paper provides a technical contribution by introducing vectorial calculus notation into a random search model, allowing the use of a multidimensional Leibniz rule. This permits a clear interpretation of the wage determination conditions. Wage determination is the main strategic decision of the firm. Changes in the posted wage will induce changes in the composition of *the applicants pool*, thereby changing the size and composition of this set. In equilibrium we know who matches whom, characterizing a set of acceptable jobs for each worker and a unique *applicants pool* for each firm. One novelty of our takeaway, is that sorting is not homogeneous in the economy. Matching and sorting are distribution dependent, and fully characterized for each point in the supports of the skills endowments and skills requirements distributions.

Matching and sorting have long been of interest to economists. Two sided matching markets (Gale and Shapley, 1962) and their implications for on sorting and stability have been well studied under various settings. In labor economics, multidimensional matching markets and sorting has been tackled only very recently (Galichon, 2018; Chiappori et al., 2016). Using the notion of multidimensional skill endowments and requirements, based on the results of the optimal transport problem (Villani, 2008), Lindenlaub (2017) presents how the production technology determines sorting in the presence of multiple dimensions, and under which specifications the equilibrium allocation and wages exist. These compelling results were rapidly incorporated into the random search model (Mortensen and Pissarides, 1994) to characterize a labor market equilibrium in multiple heterogeneous dimensions by Lindenlaub and Postel-Vinay (2016). The paper is comprehensive on the possible wage mechanisms. It characterizes the equilibrium using a whole range of wage settings: sequential auction, sequential auction with bargaining, Nash bargaining (surplus splitting), and wage posting as in Burdett and Mortensen (1998). Both papers have two things in common: sorting in both papers results from some technical characteristics of the production function and the distributions of the multidimensional vectors. In such an environment, wages and mobility depend on the surplus function (match productivity adjusted by the individual's outside option). Sorting and its sign are then an endogenous result independent of workers' and firms' preferences, a practical and powerful result.

One of the challenges that policy makers face is how to use economic models for real life policy analysis. For example, generalizations like Nash wage bargaining, in which the wage is a function of both firm requirements and worker endowments, imply that potentially all workers can perform any job. In many cases this is not sufficient if we want to know the horizontal employability of workers (how they change work across occupations or sectors). This paper aims to construct a model in which for any point in the distribution of skills endowments, one can determine a specific set of jobs that are compatible with that vector of skills. This feature is very informative in policy analysis for the case of skills retraining, workforce adaptation, and other policies aimed at reallocating the workforce. Equivalently for firms, the model allows us to characterize the segment of workers who would be willing to work for the firm at any given wage. The model then presents a new perspective for analyzing the equilibrium in labor market beyond the individual relationship, considering the *segment* of each joint distribution specified along all the dimensions that are important to workers.

Although the resulting equilibrium can be expressed in a surprisingly parsimonious fashion , it takes a high-level view of the essential heterogeneity of agents on both sides of the market and the implications that it has for decision making. The conditions imposed on the production function duly determine sorting behavior and mismatch. In contrast to previous work, this paper deals with a setting in which sorting is not homogeneous in the economy, and in which the distribution of attributes and workers' and firms' preferences define the mismatch outcome at the individual level. This paper models the microeconomic mechanism underlying mismatch in a setting that allows us to understand matching and sorting and provide an economic narrative to understand what is behind the technical properties of the equilibrium solution.

The value of such an approach is twofold: first, it complements our understanding of matching and sorting. It unveils the mechanisms that underlie technical assumptions, creating a narrative that allows one to gain a better understanding of such phenomena. Second, it provides a framework to consider sorting and matching at a more granular level, in which mismatch is a microeconomic feature.

The paper relies on a theoretical model to develop such a narrative. The starting point is the random search model of Mortensen and Pissarides (1994). In this setting, we assume that workers are willing to accept an offer only if the posted wage compensates for the utility cost of mismatch. Firms know this, and along with full information on the technology and worker type distributions, they determine the set of workers willing to accept a job offer for a given wage level. This determines a set we call *the applicants pool*. The worker's choice behaviour introduces heterogeneity in reservation wages through mismatch, in which the existence of a cost that affects utility makes it undesirable to work in a job where skills are distant from requirements. The previous literature on monopsony has used a similar mechanism, as when Bhaskar et al. (2002) and Manning (2003) introduced commuting cost into preferences, creating heterogeneity depending on workers' distance to the firm. In their models, firms compensate the average worker, so a worker who travels very long distances is not adequately compensated and decides to turn down the work. This paper's approach is conceptually similar, since the firm posts a single wage that maximizes the expected profit over workers (endogenizing who would be willing to accept the wage offer), so workers who have the highest mismatch cost in this set are worst off. As in Manning's article, this gives market power to firms and allows them to segment the market.

Firms internalize workers' behavior and use this information for optimal wage determination. Changes in the posted wage will induce changes in the composition of *the applicants pool*, thereby changing the quantity and quality[1] of this set, impacting expected earnings and productivity.

This market segmentation property of the firm's optimal strategy has links to a long-standing theme in the economic literature. In a seminal industrial organization paper, Spence (1975) discusses how a monopoly must choose not only price and quantity, but also has other instruments to select, such as quality. When it includes another instrument such as quality, the model leads to multiple equilibria. This paper relates to Spence's idea in its setting: the firm tries to maximize profits by choosing a price which affects two other features, the quantity (demand) and quality of workers it can hire. We can draw a parallel between Spence's paper and the proposed approach of this paper. In this paper, the price is the posted wage, the quantity is the size of the applicants pool, and the quality is the composition of such a

[1] It should be noted that the a given *applicant pool* will be judged to be of higher or lower quality depending on the skill requirements of the firm, so there is no absolute measure of "quality" in this sense.

pool. Like in Spence's paper, such features will determine the expected profit of each locally monopsonistic firm. There exists, however, a key difference when considering both models: in the Spence model, price does not determine quality, while in this paper, both the size and quality (composition) of the applicant pool will depend on the selected wage. This idea has recently been explored in a multidimensional setting in a series of papers on optimal platform and product design (Veiga and Weyl, 2012, 2016; Veiga, Weyl and White, 2017). Their approach influences substantially the modelling strategy presented here in two ways: first, we use their notation, based on vectorial calculus, which we adapt for the search and matching environment. Like them, this allows us to calculate the derivative under the integral sign in a compact way. The second and more valuable contribution is that we consider market segmentation and analyze the marginal acceptance set. As a result, we can study how these market segmentation ideas lead to a behavioral mechanism that implies sorting without imposing restrictions on the production technology.

A key characteristic of the proposed setting is that wages are posted with commitment, even if firms are uncertain about the worker type with which they will match. This type of framework has been used before under direct search (Galenianos and Kircher, 2009) and seems natural in many settings. First, consider a situation in which the firm needs to comply with laws or conventions concerning equality and non-discrimination. In this case, the firm may not wish, or be able to, vary its wage ex-post as a function on its potential hire. A second case could arise when the dimensions along which applicants vary are (at least partially) unobserved by the firm prior to hiring. This case is widespread and particularly relevant in online labor markets (HIT[2] and freelance markets). In such markets, the employer does not know workers' quality and makes an offer to execute a specific task. This kind of behavior has been linked to monopsony, mostly due to concentration (Dube, Jacobs, Naidu and Suri, 2018).

The idea that posted wages could affect the scope of search has been experimentally tested

[2]HIT is an acronym for Human Intelligence Task. Evidence is beginning to emerge that this kind of behavior is also present in crowdsourcing job markets (Kingsley, Gray and Suri, 2014).

recently in a directed search framework (Belot, Kircher and Muller, 2019). The role of posted wages has also been described in various theoretical papers: Moen (1997) demonstrates that when firms advertise a vacancy along with the offered wage, this leads to the competitive search equilibrium. Burdett et al. (2001) also model a framework in which a firm posts a unique price to attract buyers, and consider the strategic interaction of buyers and firms. In Galenianos and Kircher (2009) the role of posting wages is the same as in direct search models, in that prices guide search behaviour. In this model, we introduce posted wages into a random search model, enriching the random search framework with a strategic mechanism for the firms. This assumption seems appropriate for modern labor markets, in which there is evidence that employers have the power to set wages without bargaining, especially when hiring from unemployment (Hall and Krueger, 2012; Brenzel et al., 2014).

The paper is structured as follows: Section 1 details the theoretical model, describing behaviours on both sides of the market, deriving the optimal wage posting strategy and characterizing multidimensional mismatch in equilibrium. Section 2 presents the data and section 3 presents the estimation method and the results. Section 4 concludes and presents some final considerations.

1 The Model

1.1 Basic setup

The environment of the model is composed of two types of forward-looking agents, firms and workers. All agents discount the future at a common rate of r. There is a continuum of workers and firms. Workers are endowed with skills that they can offer to the market, and firms have requirements specific to their production technology. A worker is an agent endowed with a set of skills[3] that characterize its type, represented by a vector of size k.

[3]The vector of worker endowments could contain multiple dimensions of workers' characteristics, such as demographics (age, gender, race, schooling), qualifications (skills, abilities, technologies, work values), or preferences (flexibility on schedule, or importance of remote work) as long as the characteristics affect firm

The skill bundle of worker i is described $\boldsymbol{\theta}_i \in \mathbb{R}^K$ with the elements of $\boldsymbol{\theta}_i$ denoted as $\boldsymbol{\theta}_i = (\theta_1^i, ..., \theta_K^i)$. Each skill has a known support, $\theta_k \in (\underline{\theta}_k, \bar{\theta}_k)$. Each firm j is characterized by a vector of requirements $\mathbf{r}_j = (r_1^j, ..., r_l^j)$. In this paper, we will refer to firms and occupations interchangeably. Each firm is characterized by a technology that combines the requirements and worker skills in production $m(\mathbf{r}_j, \boldsymbol{\theta}_i) = m_j(\boldsymbol{\theta}_i)$ $s.t.$ $m : \boldsymbol{\theta} \times \mathbf{r} \rightarrow \mathbb{R}_+$, that produces a unique homogeneous good. In this setting, neither firms or workers can exchange or trade the vector of endowments or requirements. A probability density function $f(\boldsymbol{\theta}) : \mathbb{R}^K \rightarrow \mathbb{R}$ characterizes the distribution of workers skills in the population; a probability density function $\gamma(\mathbf{r})$ characterizes the distribution of firm skills requirements. Workers and firms know the densities. These functions are C^2, there are no mass points, and have finite moments.

The model is framed as a random search model. Agents maximize the income they receive. In unemployment, workers face a search cost $b(x) = -\bar{b}$, which for simplicity is constant[4]. This cost represents, for example, the monthly fee of usage of a job board and embeds the cost or stigma of being unemployed. Once the cost is paid, the agent will start to receive offers. The arrival rate denoted by λ is the probability by unit of time of receiving an offer. An employment relation ends at a constant exogenous rate, and thus separations are modeled with a constant risk of η.

Firms make offers to unemployed workers[5], sampled at random from a sampling distribution $s(\mathbf{r}_j)$. We will refer to such offers as *posted vacancies*. Each posted vacancy contains two pieces of information: the requirements of the firm j that makes the offer, and a proposed wage. A posted vacancy for occupation j is then a pair $(\mathbf{r}_j, w_j)$.

One of the main assumptions of the model is that wages are set ex-ante, and each firm

productivity and are embodied in the worker. The presentation of the model purely in terms of skills should thus be thought of as a simplification ease of exposition.

[4]We extrapolate from the participation decision and only consider workers who are active on the labor market.

[5]In this setting, we do not consider job to job mobility since the scope of the paper is to present a new wage-setting mechanism. The proposed setting can be enriched, including job to job mobility in the traditional posted wage form (Burdett and Mortensen, 1998) or through renegotiation (sequential auction) (Postel–Vinay and Robin, 2002), but such an extension is beyond the scope of this paper.

can post only one wage, committing to it *independently of the characteristics of the worker sampled*. This assumption differs from the partial equilibrium search literature with wage bargaining, in which there is a distribution of wages that are a direct function of workers' ability. Instead, in this setting, each firm j posts a unique wage w_j. Firms select the posted wage strategically considering the distribution of $\boldsymbol{\theta}$, since an increase in the posted wage makes the job more more attractive for a larger pool of workers, increasing the probability of a match but also potentially changing the level of expected production given the heterogeneous applicants pool. Such a mechanism suggests that firms can segment the labor market via the wage posting strategy (given their requirements), and thus have market power. Workers do not have any bargaining power, so they accept the posted wage if they decide to be employed or receive their outside option in the case they prefer unemployment.

We characterize workers' and firms' dynamic behavior in the next sections, followed by the definition of equilibrium. The next section succinctly presents some definitions, which are useful since we use vectorial calculus notation.

1.2 Definitions

We begin defining the instantaneous utility of worker i at time t when employed at occupation j. It is given by:

$$U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) = w_j - c(\mathbf{r}_j, \boldsymbol{\theta}_i)$$

Where $c(\mathbf{r}_j; \boldsymbol{\theta}_i)$ is a distance function that represents the cost of being mismatched. The cost arises, for example, by a distaste of performing the task for which one's skill does not coincide. Since we are in a model of posted wages and do not allow for a menu of wages, the cost function is the only element that incorporates mismatch into utility[6].

One of the mechanisms that is important in our setting is the worker's willingness to

[6] In other wage-setting mechanisms, wages can compensate for such distaste because they are a function of worker type.

participate in certain occupations. We denote the set of all workers willing to participate in a job j as the *applicants pool*. When workers consider a posted vacancy $(\mathbf{r}_j, w_j)$, they can calculate their instantaneous utility if they were to be employed in occupation j, and compare it to the sure outside option (unemployment). A worker is willing to join the *applicants pool* whenever the instantaneous utility is larger or equal to the outside option.

$$
\begin{aligned}
U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) &\geq -\bar{b} \\
U(\mathbf{r}_j, w_j; \boldsymbol{\theta}_i) + \bar{b} &\geq 0 \\
\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}_i) &\geq 0
\end{aligned}
$$

This determination takes into consideration only the static information of each period and assumes that workers can distinguish correctly the requirements and the wages proposed by the firm j.

Definition 1 : *The applicants pool (AP) is the set of workers that choose to participate in the market for occupation j. This set is represented as:*

$$\boldsymbol{\Theta}_j \equiv \{\boldsymbol{\theta} : \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0\}$$

Definition 2 : *The marginal applicants pool (MAP) is the set of workers who are indifferent between working or not in a job in occupation j. For them, the instantaneous utility is equal to the outside option. They are in the boundary of the set and are represented by:*

$$\partial\boldsymbol{\Theta}_j \equiv \{\boldsymbol{\theta} : \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0\}$$

The number of workers in the set $\boldsymbol{\Theta}_j$ increases when the posted wage w_j increases, diminishes when $c(\cdot)$ increases and increases with unemployment stigma. Intuitively, $\boldsymbol{\Theta}_j$ is the k-dimensional space of $\boldsymbol{\theta}$ for which the acceptance is assured for a given wage value, and $\partial\boldsymbol{\Theta}_j$ is the boundary surface of such space ($k-1$ dimensions).

Theorem 1 in Appendix B presents the Divergence theorem, a commonly known result in vector calculus. We introduce some of the notation used, and provide intuition on the elements of the applicants pool: one as a volume and the other as the surface of that volume. We are going to use also the following notation: the sign $\nabla_{\boldsymbol{\theta}} H$ is the gradient of the function H with respect to the variables $\boldsymbol{\theta}$. The $\|a\|$ is the Euclidean norm of a.

Definition 3 : *Acceptance probability. The firm calculates the mass of participating individuals as:*

$$N_j \equiv \int\limits_{\boldsymbol{\Theta}_j} f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

Definition 4 : *Marginal acceptance probability. The mass of marginal acceptance individuals, is defined as:*

$$M_j \equiv \int\limits_{\partial\boldsymbol{\Theta}_j} \frac{f(\boldsymbol{\theta})}{\left\|\nabla_{\boldsymbol{\theta}} \tilde{U}\right\|} d\tau$$

M_j then captures the responsiveness of the marginal participants, all of whom have a common reservation value in the boundary. As mentioned in the introduction, these definitions are similar to the definitions presented in recent IO literature (Veiga and Weyl, 2012, 2016; Veiga et al., 2017).

Using the definitions $1-4$, we can construct the conditional operator for the AP and MAP. For any given function $Q(\boldsymbol{\theta})$, the conditional operator is defined by:

$$\mathbb{E}[Q(\boldsymbol{\theta})|\boldsymbol{\Theta}_j] = \frac{\int\limits_{\boldsymbol{\Theta}_j} Q(\boldsymbol{\theta}_j) f(\boldsymbol{\theta}_j) d\boldsymbol{\theta}_j}{N_j}$$

$$\mathbb{E}[Q(\boldsymbol{\theta})|\partial\boldsymbol{\Theta}_j] = \frac{\int\limits_{\partial\boldsymbol{\Theta}_j} \left(Q(\boldsymbol{\theta}) \frac{f(\boldsymbol{\theta})}{\left\|\nabla_{\boldsymbol{\theta}} \tilde{U}\right\|} \right) d\tau}{M_j}$$

Using these definitions we present worker and firm behaviours.

1.3 Workers

The derivation of the continuous value of unemployment V_u is standard[7].

$$V_u = \frac{1}{1 + r\Delta t} [-b\Delta t + (1 - \lambda\Delta t)V_u +$$
$$+\lambda\Delta t\mathbf{E} \max \{V_e, V_u\}] + o(\Delta t)$$

The first term inside the square brackets is the discounted value of search. The second term corresponds to not receiving a proposal and continuing in unemployment, which occurs with probability $(1 - \lambda\Delta t)$, and the third part represents what occurs when an offer arrives and the individual chooses among two possibilities, taking the job or leaving it. The last term is a negligible component of order $o(\Delta t)$ that comes from the Bellman optimality principle. After some rearranging and passing the limit of Δt to 0, the latter equation is then:

$$rV_u = -b + \lambda\mathbf{E}_{\{\tilde{U}(\cdot)\geq 0|w_j;\boldsymbol{\theta}\}}\{V_e - V_u\} \tag{1}$$

We use $\tilde{U}(\cdot)$ instead $\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})$ to improve readability.

The value of being employed is given by the discounted value of flow utility while working, the value of employment continuation and the value of separation and going in to unemployment.

$$V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) = \frac{1}{1 + r\Delta t} [(w_j\Delta t - c(\mathbf{r}_j, \boldsymbol{\theta})\Delta t) +$$
$$+(1 - \eta\Delta t)V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) + \eta\Delta t V_u] + o(\Delta t)$$

After some algebra the value of being employed yields:

$$V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) = \frac{(w_j - c(\mathbf{r}_j, \boldsymbol{\theta})) + \eta V_u}{(r + \eta)} \tag{2}$$

[7] The reader can find the detailed derivation of all equations in appendix B

Workers will decide to take an offer if the value of employment is larger than or equal to the value of unemployment, so when $V_e(\mathbf{r}_j, w_j, \boldsymbol{\theta}) \geq V_u$.

Workers face two decisions that rely on different information sets. Unemployed workers choose to belong to the *applicants pool* (AP) considering only static information; i.e. they only consider the value of the instantaneous utility relative to the outside option. However, unemployed workers that receive the offer from firm j make their decision considering the dynamics, so the expected value of taking the job to search continuation. Given that each of the decisions is taken using a different information set, there are individuals that decide to participate in the the pool of applicants but not will accept a contract proposal, even though the information sets are related.

1.4 Firms

Let the constant returns to scale matching function $M(u, v)$ describe the technology that matches unemployed workers to vacancies. We define $M(u, v)/v = q(\omega)$ as the rate at which a vacant job is matched with an unemployed worker. The rate is dependent on the tightness in the market. We make the standard assumptions with respect to the matching function: $M(u, v)$ has constant returns to scale, $q(\omega)$ is decreasing in ω, and the $\lim_{\omega \longrightarrow 0} q(\omega) = \infty$, and that the $\lim_{\omega \longrightarrow \infty} q(\omega) = 0$. Firms are rational and maximize current and future profits. The present value of a vacant job Υ_j is formed by the discounted cost of posting the vacancy k, the continued value of maintaining the vacancy open, and the probability that a match is made times the larger value of the value of the filled job and the value of the unfilled vacancy, conditional on the person contacted being in the acceptance set.

$$\Upsilon_j = \frac{1}{1 + r\Delta t}\Bigg[-k\Delta t + (1 - q(\omega)\Delta t)\Upsilon_j + \\ + q(\omega)\Delta t \int_{\boldsymbol{\Theta}_j} \max\{J(\mathbf{r}_j, \boldsymbol{\theta}), \Upsilon_j\} d\boldsymbol{\theta}\Bigg] + o(\Delta t)$$

After some manipulation the flow value of a vacant job is written as:

$$r\Upsilon_j = -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j, \boldsymbol{\theta}) - \Upsilon_j\}|\boldsymbol{\Theta}_j\right] \tag{3}$$

where the conditional expected value with respect to firm j's AP is multiplied by the acceptance probability and the matching rate.

We define the flow value of a filled job with $J(\cdot)$, composed of the present value of flow profits of the match for the firm, its continuation and its termination. The flow value of a filled job will depend on firm requirements, worker endowments, and the posted wage.

$$J(\mathbf{r}_j, \boldsymbol{\theta}) = \frac{[m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j]\Delta t}{1 + r\Delta t} + \frac{(1 - \eta\Delta t)}{1 + r\Delta t}J(\mathbf{r}_j, \boldsymbol{\theta}) + \frac{\eta\Delta t}{1 + r\Delta t}\Upsilon_j + o(\Delta t)$$

which after some algebra yields:

$$J(\mathbf{r}_j, \boldsymbol{\theta}) = \frac{m_j(\boldsymbol{\theta}) - w_j + \eta(\Upsilon_j)}{(r + \eta)} \tag{4}$$

In equilibrium in the Mortensen and Pissarides (1994) framework, a firm accepts any match for which the value of the filled vacancy is larger than or equal to the discounted value of the vacancy $J(\mathbf{r}_j, \boldsymbol{\theta}) \geq r\Upsilon_j$ (hence the max in equation 3). The firm also faces a strategic decision on wage-setting since the posted wage determines the flow value of a vacant job and the flow value of a filled job, as well as the applicant pool. Inserting (4) into (3) we obtain the flow value of an unfilled job conditional on the posted wage. Equation (5) describes how the posted wage w_j might change the value of an unfilled job.

$$r\Upsilon_j = \frac{-k(\eta + r) + q(\omega)N_j\mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j]|\,\boldsymbol{\Theta}_j\right]}{(\eta + r) + N_j q(\omega)} \tag{5}$$

Equation 5 shows that the posted wage is the only decision variable available to the firm that can affect the flow value of an unfilled vacancy. An increase in the posted wage will reconfigure the applicant pool, increasing the applicants pool size and thus increasing the acceptance probability N_j. The increase is also induces a change in the expected flow net value of the match. The sign and magnitude of such change will depend on the technology and its sensitivity to the changes in the composition of the applicant pool, and it will also depend on the characteristics of the population and its distribution (on the shape of the distribution of types $f(\boldsymbol{\theta})$).

1.5 Equilibrium & Wage-setting

1.5.1 Balanced flows and worker densities

In equilibrium, the flows are balanced between states, the number of posted vacancies is optimal for each firm, and the posted wage is optimal. In this section we present the conditions under which such an equilibrium exists, emphasizing how the wage-setting mechanism responds to a behavioral optimal response in equilibrium.

For the aggregate flows to be balanced in the steady state, the flow into employment is equal to the flow out of unemployment. We assumed that an employment relation has an exogenous constant rate of termination η, and that the probability per unit of time of being matched is λ, such that unemployed meetings are equal to employed terminations, $\lambda u = \eta(1-u)$. Solving for u we have:

$$u = \frac{\eta}{\lambda + \eta} \tag{6}$$

For the distributions of skill endowments of unemployed workers to be stationary we know that outflow of of workers of each type must be equal to the inflow. The outflow of workers from the firm is defined by the share of employed workers in the firm by skill type, multiplied by the rate of termination of the contracts $\eta(1-u)\ell(\boldsymbol{\theta}, \mathbf{r}_j)$, where $\ell(\boldsymbol{\theta}, \mathbf{r})$ is the density of workers with skill bundle $\boldsymbol{\theta}$ employed at firm $\mathbf{r}_j$. The inflow of workers into employment must be equal to the arrival of unemployed workers per type, adjusted by the probability of sampling, $u\lambda f(\boldsymbol{\theta})s(\boldsymbol{r}_j)$. By equalizing the inflows and outflows and using equation 6, we have that the density of workers with skill bundle $\boldsymbol{\theta}$ employed at firm $\mathbf{r}_j$, unconditional and conditional on the AP, is given by:

$$\ell(\boldsymbol{\theta}, \mathbf{r}_j) = f(\boldsymbol{\theta})s(\boldsymbol{r}_j) \tag{7}$$

$$\ell(\mathbf{r}_j) = \ell(\boldsymbol{\theta}, \mathbf{r}_j | \boldsymbol{\Theta}_j) = \int_{\boldsymbol{\Theta}_j} f(\boldsymbol{\theta})s(\boldsymbol{r}_j)d\boldsymbol{\theta} = N_j s(\boldsymbol{r}_j) \tag{8}$$

With these equations we can calculate the search intensity $\frac{s(\boldsymbol{r}_j)}{\gamma(\mathbf{r}_j)}$, and firm size $\frac{\ell(\mathbf{r}_j)}{\gamma(\mathbf{r}_j)}$ for each of the firms in the economy. Those values depend on the AP, whose composition is determined completely by the posted wage w_j. We can use equation 5 to determine the missing components of the equilibrium, the optimal vacancies per firm, and the optimal wage posting. To retrieve the optimal vacancies per firm we use the free entry condition $r\Upsilon_j = 0$, while for the definition of the optimal wage we use the profit maximization condition $\frac{\partial r\Pi_j}{\partial w_j} = 0$.

The calculation of the last derivative presents some complications since the posted wage is in the integration limit. In order to calculate such values, we use the Leibniz rule to obtain the equilibrium of the model.

1.5.2 Profit maximization, wage determination and equilibrium

The value of an unfilled job, conditional on the posted wage, is presented in equation 5. From this equation we derive two of the equilibrium conditions for which the employer assures that the posted wage is optimal. The first condition is free entry. Imposing free entry implies that the number of vacancies in the market is endogenous to the model, so that firms can make no additional profit by posting an additional vacancy. As can be seen, the number of vacancies can affect the profits of the firm, as it affect $q(\omega) = q(\frac{v}{u}, 1)$ the matching function. The arrival rate of offers also is affected by the increase in the number of vacancies, and in equilibrium $\lambda(v)$ is an concave increasing function on vacancies.

$$\lambda = \frac{q(v,u)}{u} = q\left(\frac{v}{u}, 1\right) = q(\omega) \equiv \lambda(v) \tag{9}$$

Following Mortensen (1998), we impose free entry, meaning that the value of of an unfilled vacancy is zero, and insert equation 9 into equation 5. After some algebra we get:

$$kv_j = \lambda(v_j)\frac{N_j}{\eta + r}\mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j]|\,\boldsymbol{\Theta}_j\right] \tag{10}$$

Given an equilibrium posted wage $(w)^*$ equations 1-8 are well defined. Assuming Inada conditions hold and that $\lambda(v)$ is increasing and concave, equation 10 has a stable equilibrium for a positive $v_j^* > 0$.

Using the preceding condition, the optimal value of the posted wage will satisfy the boundary condition (equation 10). After taking the derivative with respect to the posted we get:

$$\frac{\partial kv_j}{\partial w_j} = \frac{\partial}{\partial w_j}\left(\lambda(v_j)\frac{N_j}{\eta + r}\mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j]|\,\boldsymbol{\Theta}_j]\right)\right) \tag{11}$$

Applying the Leibniz rule, and reorganizing terms, we get the expression for the optimal posted wage. After some manipulation, the optimal posted wage rule solves the following equation.

$$\begin{aligned} M_j\mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial\boldsymbol{\Theta}_j\right](w)^* = -N_j + \\ + M_j\mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial\boldsymbol{\Theta}_j\right]\mathbb{E}\left[m(\cdot) \mid \partial\boldsymbol{\Theta}_j\right] + \\ + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w}, m(\cdot) \mid \partial\boldsymbol{\Theta}_j\right] \end{aligned} \tag{12}$$

The optimal posted wage equation gives the posted wage that the firm commits to offer, taking into consideration the multidimensional skill distribution in the economy, the mismatch cost associated with work in a job, and the pool of candidates that will accept the offer. The decision depends on the size of the pool N_j, the sensitivity of the infra marginal workers to changes in wage and its relationship to their productivity.

It is worth noting, that we can calculate the optimal posted wage both from condition 10 and the steady state profit flow. Maximizing either equation leads to the same definition. To show this fact we derive the optimal wage from the steady state profit flow, as is done in the basic Burdett and Mortensen (1998) wage posting model.

We write profit as the expected flow value of production net of cost, calculated over the density of the employer's applicants pool.

$$\Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j) = \max_w \mathbb{E}\left[m(\cdot) - w | \boldsymbol{\Theta}_j\right] \ell(\boldsymbol{\theta}, \mathbf{r}_j | \boldsymbol{\Theta}_j)$$

Replacing the value of the joint density (eq. 8) in the above equation, we can rewrite profits in terms of the *quantity* and *quality* of the AP. In this framework, we define quality as the expected match productivity net of cost evaluated over the pool of applicants. Profits are related to quality since they depends on the distribution of skills within the applicants pool. They are related to quantity since the quality of the applicants pool is multiplied by the mass of workers that belong to the pool. The firm will select the wage that defines the best pool of candidates that will accept the offer, and by doing so it will maximize it's profits.

$$\Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j) = \max_w \underbrace{\mathbb{E}\left[m(\cdot) - w | \boldsymbol{\Theta}_j\right]}_{\text{Quality}} \underbrace{N_j}_{\text{Quantity}} s(\mathbf{r}_j) \tag{13}$$

Using the differential under the integral sign (eq. A.1), the firm chooses the posted wage that maximizes the steady-state profit flow. The first order condition of this problem is given by[8]:

[8] Derivation in the appendix

$$\frac{\partial \Pi_j(\mathbf{r}_j, \boldsymbol{\theta}_i; w_j)}{\partial w} = s(\mathbf{r}_j)\left[-N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w}(m(\cdot) - w) \mid \boldsymbol{\Theta}_j\right]\right] = 0$$

Which after some manipulation, and solving for the posted wage, yields the optimal posted wage rule.

$$\begin{aligned} M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial \boldsymbol{\Theta}_j\right](w)^* = -N_j + & \\ & + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w} \mid \partial \boldsymbol{\Theta}_j\right] \mathbb{E}\left[m(\cdot) \mid \partial \boldsymbol{\Theta}_j\right] + \\ & + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w}, m(\cdot) \mid \partial \boldsymbol{\Theta}_j\right] \end{aligned} \tag{14}$$

As mentioned before the optimal wage can be retrieved both from profit maximization or by equalizing flows, reconciling the older (Burdett and Mortensen, 1998) and newer models (Ellingsen and Rosén, 2003). With the equilibrium values v^* and w^*, the model is closed and all the equations of the model are well defined.

It is essential to note some results from the theoretical model. First, consider the size of the marginal applicants pool. We calculate expected productivity with respect to this set. Its size and composition define the participation patterns, including the firm's mismatch. An increase in the wage will increase the number of people willing to accept the job and increase the mismatch for that firm type. In equilibrium, the optimal wage posting strategy integrates such information and offers the wage that corresponds to the marginal group composition that maximizes expected marginal match productivity, adjusted for the co-variance between preferences and the production function.

Another important result from the theory comes from the role of multidimensionality. Recent literature shows that omitting multidimensionality could lead to wrong results (Lise and Postel-Vinay, 2015; Lindenlaub and Postel-Vinay, 2016). In the presented model, multidimensionality plays different roles: through mismatch, skills substitution plays an important role because firms can still substitute one skill for another. More importantly, the marginal set effects becomes relatively more important when the number of dimensions increases, which will have consequences for the proposed wage schedules.

1.6 Narrative of the model: order of events

Here we would like to emphasize the narrative behind the matching and sorting process in the proposed model. Such a narrative helps us to understand how both types of agents use the available information. In order to construct this narrative, we propose to analyze the model at two specific points in time. These points in time occur simultaneously, but we present them separately since the agents use different sets of information.

In the first moment, the firm understands the participation rule that workers have. Workers are passive at this point, in the sense that they only accept or reject based on an already defined set of acceptable postings. Recall that we have defined each posting as the combination of information about an offered wage with commitment and specific requirements. The firm internalizes how the worker processes this information and sees how different types accept and reject the offer for different posted wage levels. Using the wage rule, firms optimize the size and composition of the applicant pool. The wage corresponds to the point at which a marginal increase in the posted wages changes the composition in the applicants pool in a way that decreases the firm's steady-state profit.

The second moment is when the offer arrives and the worker decides whether or not accept the match. This part is similar to the classic interpretation. To create a narrative, we imagine that a firm randomly sends a posting (wage and requirements) to a worker sampled from the skill distribution. Workers compare the flow value of employment and unemployment to accept or reject the match, but this value depends not only in the wage but also on the requirements of the firm relative to the skill endowment of the worker. If the employment value is larger than the value of unemployment, the worker accepts the match until the match dissolves with an exogenous probability.

Comparing the information sets at these two points in time, we can observe that the choice is dynamic in the second one, while in the first, the worker chooses based on information that determines flow utility. Given that there is no job to job mobility, skills depreciation, or learning, the two sets of information are compatible and lead to the same outcome.

Following this narrative, what are the possible shocks that can modify the worker's decision? The most evident is the value of the outside option. Changes in the distribution of skills will also change the expected value of the match, the posted wage, and thus worker decisions. Changes in the production technology will also affect the posted wage, and thereby the acceptance set.

In the next sections, we present the data and estimate the model for France.

2 Data

We use the data for France from the Programme for the International Assessment of Adult Competencies (PIAAC). The survey was developed by the OECD and data collection for France was undertaken between September and November 2012. The PIAAC provides internationally comparable data about skills of the adult population in 24 countries. The sample consists of adults between 16 and 65 years of age. Even if sampling schemes are different between countries, the data provides post-sampling weightings which allows one to fit the principal moments of labor market indicators, earnings, demographics and the skills distribution. In order to match the measured skills, a multiple imputation method is proposed, and 10 plausible values are provided for both literacy and numeracy. For each plausible value a weight is also provided.

The survey includes a direct assessment of cognitive skills in two main domains: literacy and numeracy. For literacy, the survey measures the ability to understand written texts; For numeracy, it quantifies the ability to access, use, interpret, and communicate mathematical information and ideas. An optional dimension is also measured, Problem solving in technology-rich environments, which is understood as the ability to use digital technology. The latter was not measured for France, so we use only the literacy and numeracy measures. It is important to note that these measures are not self declared, but rather directly assessed through a test administrated by the interviewer.

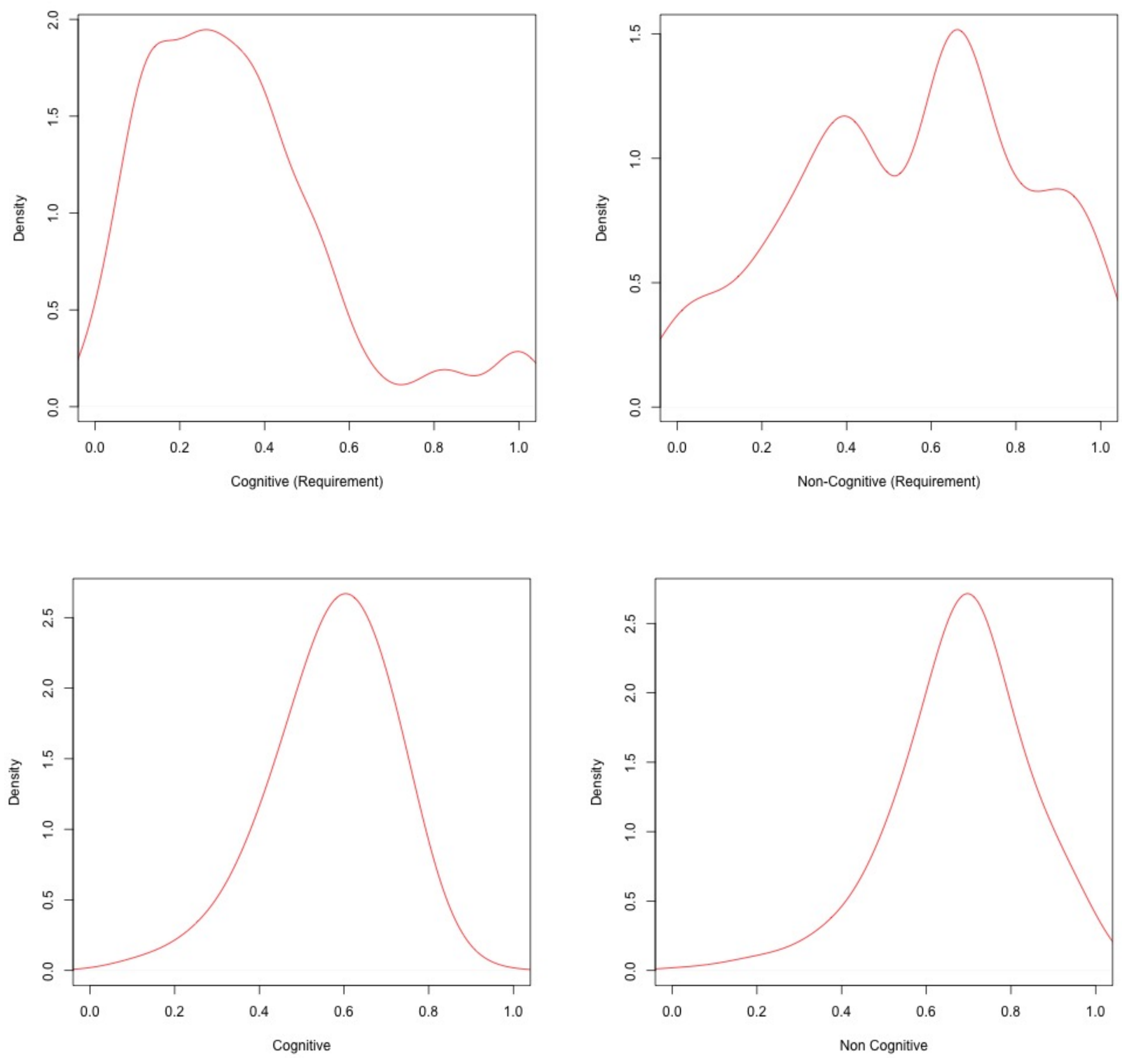
The non cognitive skills measures are derived from the answers to the background questionnaire of the survey. In this part, six questions about attitudes and interest toward learning are asked. These measures are related to personality and intelligence and can be linked to one of the big 5 personality traits: openness to experience (Goff and Ackerman, 1992).

To assess job requirements, we use O*NET data. Specifically, the O*NET is a U.S.-based system which provides up-to-date and detailed descriptors of the requirements for each occupation in terms of the knowledge, skills, and abilities required by workers, as well as how the work is performed in relation to tasks, work activities, work context and other descriptors (Onetcenter, 2016). In this paper I used the skill information on requirements and I construct two vectors of skill requirements for cognitive and non cognitive skills using factor analysis.

Figure 1 presents the cognitive and non-cognitive skills supply (top figures) and demand (bottom figures) after having applied a linear transformation to each set of measures in order to make the supports of the distributions fit in the $[0, 1]$ interval. These figures highlight the main challenge the model is designed to address: how to allocate such heterogeneous demand to the heterogeneous supply of skills, especially when the distributions have different shapes, and how do wages and preferences affect the allocation.

Table A.1 presents the moments used in the paper for the estimation. The values for the wages are calculated directly from the data.

Figure 1: Requirements and skills



3 Estimation

I estimate the model by indirect inference using the simulated method of moments (SMM). I calculate the moments of the distribution of skill endowments, requirements, the value of the unemployment and wage deciles using the survey data. These are the observed (sample) moments z^o. Using a set of proposed parameters π, we then solve the model starting from equation 14 and equation 10. We then calculate the moments $z^s(\pi)$, generated by the set of parameters π, for the simulated population. The estimation procedure minimizes the distance between the empirical observed moments and the simulated moments, so that the wage distribution, skills endowment distribution, skills requirements distribution and unemployment rate are replicated as close as possible. The minimization problem an be written as:

$$P(\hat{\pi}) = \min_{\hat{\pi}} \sum_{I} \omega_i (z_i^o - z_i^s(\pi))^2 \tag{15}$$

where ω_i is a weight that values the importance of the moment in the sample. For the actual exercise, the unemployment level and the percentiles of the wage distribution are assigned double the importance of the skill percentiles.

3.1 Parametric specification

One of this paper's main contributions is to model, in the most granular way possible, the decision of each agent. From the theoretical results, we saw that the model's main equations involve the distributions of skill endowments and requirements. The way we model these multivariate distributions will have an impact on our results. Papers that used similar estimation procedures (Lise and Postel-Vinay, 2015) combine two beta marginals for the skills and requirements distribution with a Gaussian copula.

The dependence between the dimensions is constant when using Gaussian copulas, while is likely that such dependence changes in the different parts of the distribution. We therefore model the skills and endowment joint distributions using a Frank copula, which allows the skill distribution to be non-symmetric and allows the dimensions to be locally dependent. Our Frank copula has beta marginals, allowing different shapes on the margin. This copula can also provide the LDF (Local Dependence Function), which describes the point correlation of two random variables, (x_1, x_2), at each point of the common support. In this way, we can model and understand the joint variation of the strength of association at each point of the support. The Local Dependence Function is generally interpreted as a "local Pearson correlation" and is defined as:

$$\gamma(x_1, x_2) = \frac{\partial^2 \log f(x_1, x_2)}{\partial x_1 \partial x_2}$$

If the x_1, x_2 are independent random variables, the LDF is 0 over the support of x_1 and x_2. In the case of a bivariate Gaussian, the value is constant and equal to $\frac{r}{1-r^2}$. The shape of the Frank copula is defined by:

$$F(x_1, x_2) = -\frac{1}{\bar{\alpha}} \log \left(1 + \frac{\left(e^{-\bar{\alpha} F_1(x_1)} - 1\right)\left(e^{-\bar{\alpha} F_2(x_2)} - 1\right)}{e^{-\bar{\alpha}} - 1} \right)$$

For the Frank copula the LDF is:

$$\gamma(x_1, x_2) = 2\bar{\alpha} f(x_1, x_2)$$

The estimation of the multidimensional densities is the first step of the simulation. In the results the estimation points with more mass will have a larger correlation. The sign and size of the $\bar{\alpha}$ parameter will determine the correlation in each point of the simulated grid and determines the weight assigned by the joint density to each point.

For the purposes of estimation, we can define a grid and calculate the value of all functions of the model for each point to determine the applicants pool that each firm faces, and simulate the complete model. In the simulation we use only two skills. We also need to specify functional forms for the cost utility and the production function in order to generate the remaining simulated moments.

We model the cost function as the weighted euclidean distance between the skills endowment of the worker and the skills requirements of the job. In this formulation, the weights ξ_k, which are estimated, value the mismatch cost for the individual in each dimension.

$$c(\boldsymbol{r}_j, \boldsymbol{\theta}_i) = \left(\sum_{k=1}^{2} \xi_k (\boldsymbol{\theta}_{ik} - \boldsymbol{r}_{jk})^2 \right)^{0.5}$$

Using this specification, we can calculate the flow utility function. Recall that flow utility is the difference between the posted wage and the worker's disutility from the mismatch. The function used is then given by $u(r_j, \theta_i, w_j) = w_j - c(r_j, \theta_i)$. Given a wage, we can calculate the density of the applicants pool for each firm and its marginal pool of applicants. All the operations on expected productivity and expected costs are then calculated to determine the posted wage for each requirements vector in the grid.

We also specified the production function using a constant elasticity of substitution functional form. We use this functional form since we would like to test skills complementarity in production.

$$m(\boldsymbol{r}_j, \boldsymbol{\theta}_i, \xi) = \left(\phi_c \left(\frac{\theta_i^c}{r_j^c} \right)^{\mu} + \phi_{nc} \left(\frac{\theta_i^{nc}}{r_j^{nc}} \right)^{\mu} \right)^{\frac{1}{\mu}}$$

The simulation generates work histories of individuals and firms sampled from the skill distributions and matches the observed unemployment, earnings, and skills distribution. For comparing simulated unemployment in the optimization to observed unemployment, we consider the average unemployment of the last 50 simulated periods after burning the first 100 simulated periods.

The final functional form we need to define is the matching function. One caveat of our estimation is that we do not have the duration of employment or unemployment in our data, so estimating the separation rate does not have an observed counterpart and can only be fit through its influence on other moments that are matched in the simulation. The matching function is specified by the following functional form $\lambda(v) = m(v, u) = \psi\sqrt{uv}$.

The set of parameters to estimate is then:

$$z = [\alpha_c, \alpha_{nc}, \beta_c, \beta_{nc}, \bar{\alpha}, \alpha_c^s, \alpha_{nc}^s, \beta_c^s, \beta_{nc}^s, \bar{\alpha}^s, \xi_c, \xi_{nc}, \phi_c, \phi_{nc}, \mu, \psi, \lambda, \eta, b, r]$$

In the next section we present the main results of the estimation and discuss their implications.

3.2 Results

Table 1 presents the estimation results. The first four parameters in the table determine the shape of the estimated copula's beta marginals. The fifth parameter is the Frank copula's strength correlation parameter and determines the degree of association between cognitive and non-cognitive skills endowments. This parameter is positive (1.233), implying a positive correlation, especially in the distribution's more dense parts. The next five parameters have the same interpretation, but for the distribution of the requirements. The correlation between cognitive and non-cognitive requirements is stronger (1.816). Considering both estimates together gives us a hint about how these distributions are different and points to the difficulties that the allocation mechanisms face: on the workers side, the distribution is flatter and even if positively correlated, less correlated than the distribution of the requirements. A visual representation of such copulas is presented in figure 2.

Considering the cost function parameters, workers assign a higher disutility to being mismatched in cognitive skills than non cognitive skills. When we consider the firm side, we can see the production technology assigns higher weights to non-cognitive skills. We also find that the elasticity of both skills in the production function is negative, even if small. This result suggests that there is a degree of complementarity in production between cognitive and non-cognitive skills.

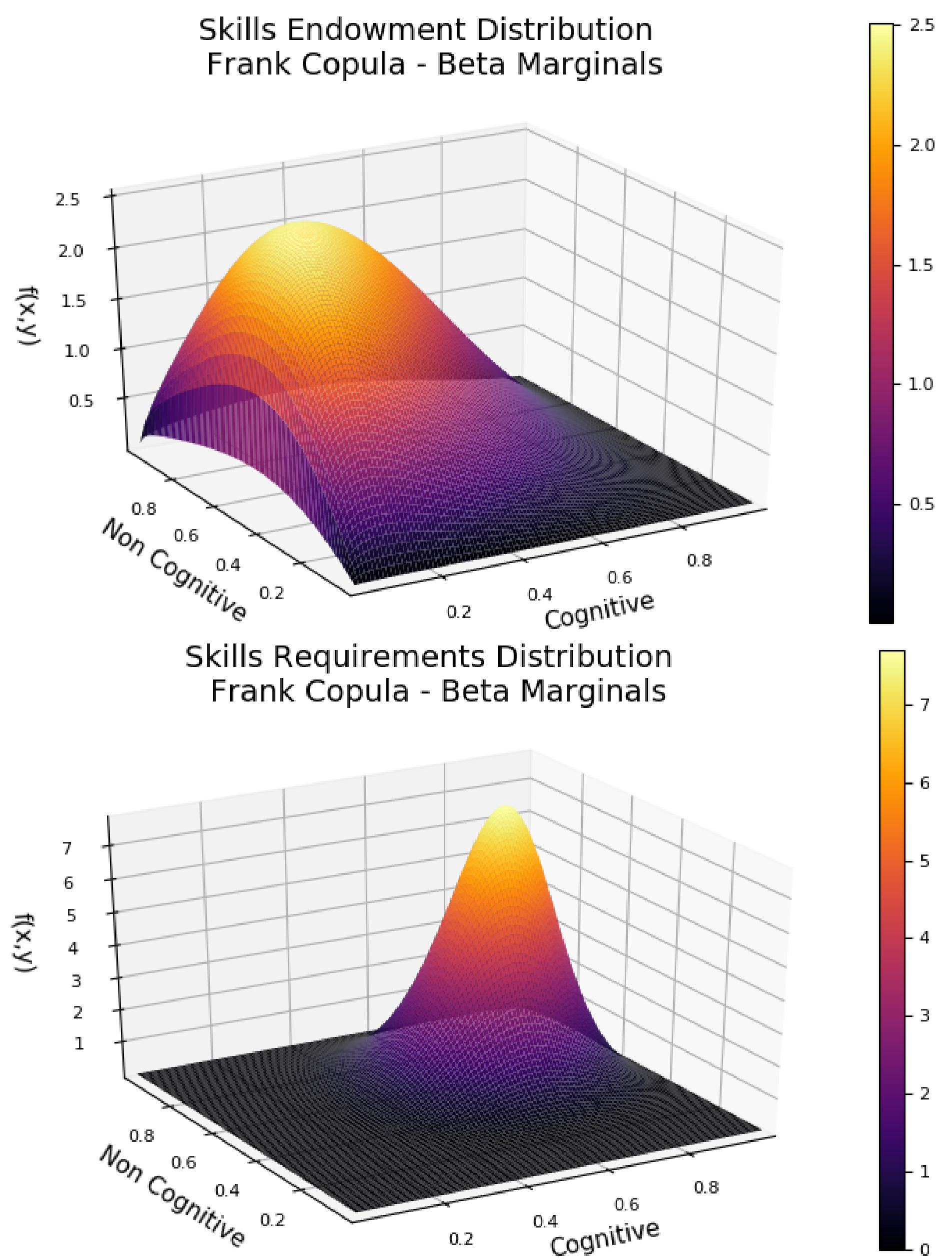
The next part of the table shows the estimated labor market parameters: the matching

Table 1: Estimated parameters

	Estimate	C.I. (95%)	Description
α_c	1.360	$[1.357, 1.364]$	Shape parameter from the beta distribution, Cognitive skill endowment.
α_{nc}	1.971	$[1.969, 1.973]$	Shape parameter from the beta distribution, Non cognitive skill endowment.
β_c	2.618	$[2.616, 2.620]$	Shape parameter from the beta distribution, Cognitive skill endowment.
β_{nc}	1.416	$[1.414, 1.418]$	Shape parameter from the beta distribution, Non cognitive skill endowment.
$\bar{\alpha}$	1.233	$[1.226, 1.238]$	Strengh of correlation parameter.
α_c^s	6.790	$[6.784, 6.797]$	Shape parameter from the beta distribution, Cognitive skill requirement.
α_{nc}^s	6.196	$[6.194, 6.198]$	Shape parameter from the beta distribution, Non cognitive skill requirement.
β_c^s	2.978	$[2.975, 2.981]$	Shape parameter from the beta distribution, Cognitive skill requirement.
β_{nc}^s	3.464	$[3.146, 3.648]$	Shape parameter from the beta distribution, Non cognitive skill requirement.
$\bar{\alpha}^s$	1.816	$[1.812, 1.821]$	Strengh of correlation parameter.
ξ_{nc}	90.71	$[86.40, 94.12]$	Cost function weight, Non Cognitive
ξ_c	105.31	$[97.22, 112.91]$	Cost function weight, Cognitive
ϕ_{nc}	335.12	$[333.09, 338.14]$	Production function weight, Non Cognitive
ϕ_c	311.35	$[308.04, 315.58]$	Production function weight, Cognitive
μ	-0.429	$[-0.430, -0.428]$	Production function elasticity
ψ	0.542	$[0.541, 0.544]$	Matching function parameter
λ	0.055	$[0.040, 0.063]$	Finding rate - Offer arrival
η	0.006	$[0.006, 0.006]$	Separation rate
b	250		Unemployment stigma (Calibrated)
r	0.004		Discount rate (Calibrated)
k	$0.6E[m(r.\theta)]$		Cost of a vacancy (Calibrated)

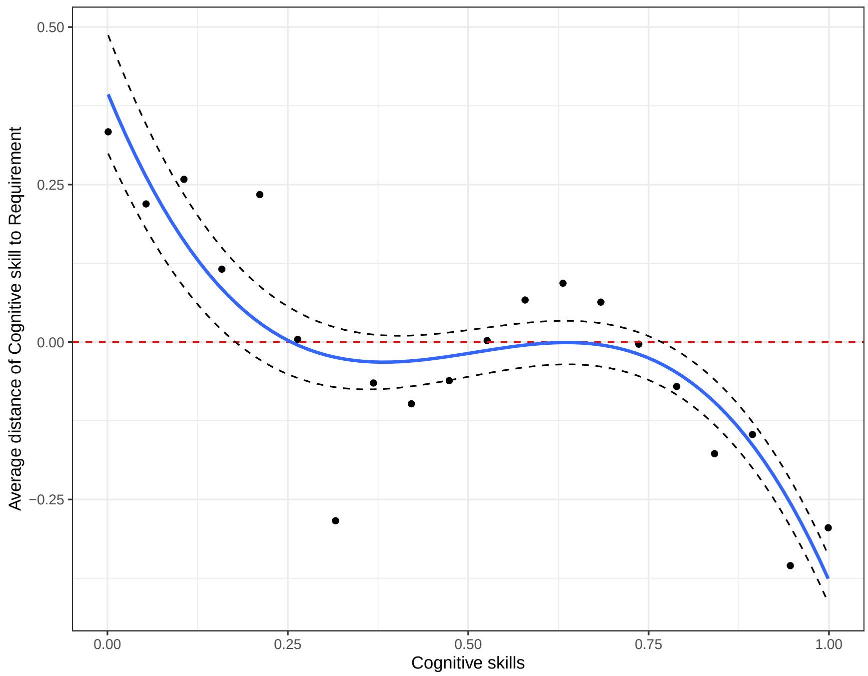
function scale parameter, arrival rate and the exogenous separation rate. We can see that the values of the finding rate are higher than the usually in the reported literature. This is because, in our model, the arrival of an offer does not imply acceptance. An offer from a firm can arrive at a worker outside its applicants pool. In this way, the mismatch costs creates additional frictions which do not depend on the traditional parameters.

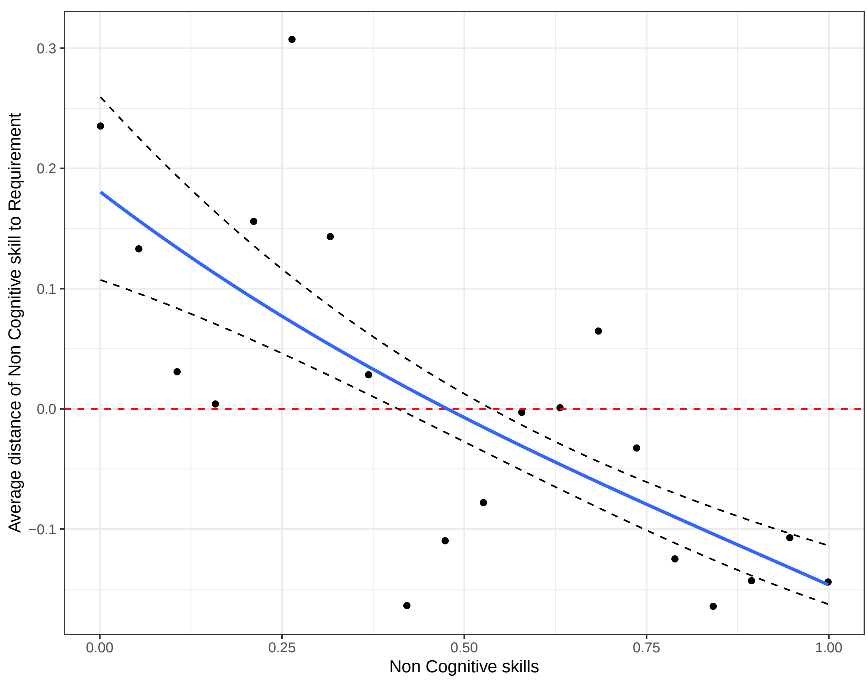
Figure 2: Copula requirements and skills



Using our estimates, we can recover the mismatch across the support of worker's skill

Figure 3: Average difference between skills by type





Panels (a) and (b) display the 3^{rd} order polynomial interpolation of the average difference between the requirements and endowments along the support of each type of skill. Panel (a) presents the information for cognitive skills, while panel (b) for non cognitive skills.

distributions from the simulation results. Figure 3 presents the difference between the requirements and the skills endowments over the support of the distribution of cognitive and non-cognitive skill endowments. We can observe a clear decreasing pattern in the figures: low skill workers are underqualified and high skill workers are overqualified. In the first part (until the 25^{th} percentile), workers are underskilled in cognitive skills. In the last part of the distribution (after the 75^{th} percentile), they are over-skilled. In the middle of the distribution, we can not reject the null hypotheses of the absence of mismatch, in some cases positive, while negative in others. This slope is by construction negative, since there can be no values in the requirements distribution below the smallest value of compact support of cognitive skills endowment distribution (hence a positive difference) and no values in the requirements distribution above the largest value in the support of the endowments distribution (hence a negative difference). Nevertheless, it is worth noting the share of the population correctly matched in each of the dimensions, and how this result can be inferred from the estimated distribution of skills and requirements. The lower panel of the figure reveals the same decreasing pattern of mismatch across the whole distribution. Even if the degree of mismatch is quantitatively lower than in the previous case, we can see that the share of workers for whom we cannot reject the null of no mismatch is much smaller, with only about 10 percent of workers being correctly matched on the non cognitive skills dimension.

3.3 Skills importance in wage determination:

The wage posting strategy derived here can be used to characterize the wage at each point of the support of the multivariate distribution. To do this we need to know the primitive parameters of preferences and the production function, which can only be recovered via structural estimation. What would happen if, in the absence of such parameters, we estimated directly a reduced form regression the effect of skills on wages?

To answer this question we estimate a simple OLS regression on the simulated data using the estimated data generating process of the form:

$$Y_i = \beta X_i + \epsilon_i \tag{16}$$

where Y_i represents the wage of individual i in our simulated sample. The explanatory variables are the different skills measures available in the career simulation: first the worker skills, second the skill requirements, and last the mismatch. The sample in consideration is last simulated period. We use this cross-section to perform our regressions, since is similar to the databases from which researchers typically calculate their estimates. We estimate the model for each of the skills available.

Table 2: Contributions of skills endowments and requirements to wages

	$\hat{w}$ - Simulated Wage		
	(1)	(2)	(3)
Cognitive Skill	6.347^{***} (0.4574)		
Non Cognitive Skill	4.267^{***} (0.4523)		
Cognitive Requirement		10.259^{**} (4.1615)	
Non Cognitive Requirement		13.909^{***} (0.9974)	
Cognitive mismatch			-0.714 (0.6128)
Non Cognitive mismatch			-0.922^{***} (0.1477)
R^2	0.140	0.187	0.061

Note: $*\ \ P < 0.1; **\ \ P < 0.05; ***\ \ P < 0.01$

Table 2 presents the results of the different estimations. In the first column, worker skills are regressed on the wage. The weights of the cognitive skills are 50% higher than the non-cognitive skills. Nevertheless, the explanatory power of the model using worker skills is lower than the one that uses skill requirements. When considering skill requirements, we can not distinguish between the effect of cognitive and non cognitive, but the explanatory power is the highest. These two findings could be explained by the shapes of the distributions (see figure 2), as the joint density of worker skills is flatter and covers the whole support, while the the requirement distribution is concentrated in higher values. The last column in Table 2 presents the contribution of mismatch to wages. The effect of cognitive mismatch is not significant, while the non cognitive mismatch is significant. This could be reflecting the fact that we estimate that a smaller share of workers are actually mismatched on the cognitive skill dimension than the non cognitive dimension, and that skills mismatch on the non cognitive dimension is more penalizing for firms than on the cognitive dimension. Both values are small, in comparison to the regressions when we use worker skills and skills requirements.

These results highlight the importance of accounting for mismatch in understanding the role of skills in wage determination. They suggest that a linear model that does not take into consideration the complete dynamic of the wage rule, would be unable to reproduce important sources of wage variation. In particular, neglecting the skills of either side of the employment relation ends up estimating only an average effect over the whole distribution, omitting the importance of firms' ability to segment the market. Even if the values are significant, they are unable to capture the rich story behind the way workers and firms match when skills are multidimensional.

4 Conclusion

In this paper, we provide a micro founded model of matching, sorting, and mismatch in a random search environment in which we introduce multidimensional heterogeneity of worker endowments and firm requirements. We provide a microeconomic narrative that extends our understanding of the matching and sorting process, adapted to a setting in which heterogeneous and multidimensional skill endowments and requirements characterize agents. In this setting, firms post a wage with commitment, independent of the type of workers that accept it. Wage setting becomes a strategic decision and the optimal wage rule considers the distribution of the types, the complementarity of skills in the production function, and the size and composition of the set of workers willing to accept the job. An increase in the posted wage increases the applicants pool size but might increase mismatch depending on the endowment distribution and preferences. We derive the wage rule both from the steady-state profits and the flow value of an unfilled job conditional on posted wage. From both equations, we get an equivalent result.

We then estimate the model for France. We find that the correlation between skills endowments is lower than the correlation for requirements. We also find that cognitive and non-cognitive skills are weak complements in production, which makes mismatch more costly. The estimation allows us to calculate mismatch at a granular level. When we analyze the degree of mismatch along the distribution of cognitive and non-cognitive skills endowments, we observe that the intensity of mismatch is larger for non cognitive skills than cognitive skills for the case of France. Good matches occur for the middle two quartiles of the cognitive skills distribution, while they are closer to the 10 percent of workers closer to the median for non cognitive skills distribution.

In sum, multidimensionality of skills plays an important role in matching and wage determination. Multidimensionality can aggravate problems of mismatch since cognitive and non cognitive skills are found to be complements in production and this affect wages. Finally because mismatch is costly to workers, it represent an additional friction in the labor market. The estimated offer arrival rate must therefore increase to compensate for such inefficiency.

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A Appendix I

In order to consider the margin and the interior solution, we will use a multidimensional version of differentiation under the integral, or the Leibniz Rule. This approach is well known in mathematics and physics (Flanders, 1973; Dieudonné, 1959), and recently has been used also in other fields of economics (Veiga and Weyl, 2012, 2016; Veiga et al., 2017)[9].

Definition : ***(Leibniz Rule)*** *Consider the function:*

$$G(w_j) = \int_{\boldsymbol{\Theta}:\tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} g(\boldsymbol{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

Differentiating this function with respect to w_j yields:

$$\begin{aligned} \frac{dG(w_j)}{dw_j} &= \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) \right) d\boldsymbol{\theta} + \\ &+ \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0} \left(\frac{\partial \tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\|} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) \right) d\tau \end{aligned} \tag{A.1}$$

The effect of a change in wages, following equation A.1, can be divided in two parts:

- The first part is the average effect in the acceptance set with respect to wages and implies a transfer of the match productivity. It measures how the function is sensitive to changes in wages for a given set of multidimensional requirements.

- The second part is the effect on the boundary, and measures the effect on marginal individuals to accept or not the proposed offer, proportional to the marginal reservation of participation.

Is important to remark that under this setting, the maximization is in terms of the marginal profitable accepted worker, and the wage is the instrument that the firm has to

[9] See derivation 7 in the appendix.

segment the pool of unemployed to maximize profits. We follow the equilibrium description, presenting the results from the free entry condition and number of vacancies, and optimal wage.

B Appendix II

Theorem 1 (Divergence theorem) *We assume that $\boldsymbol{\Theta}_j$ is a compact domain of integration with a piecewise smooth boundary $\partial\boldsymbol{\Theta}$. If there is a function $\boldsymbol{H}$ that is continuous and differentiable vector field defined on the boundary of $\boldsymbol{\Theta}_j$, we then have:*

$$\int_{\underline{\theta}_K}^{\bar{\theta}_K} \cdots \int_{\underline{\theta}_1}^{\bar{\theta}_1} (\nabla \cdot \boldsymbol{H}) d\theta_1 ... d\theta_K = \int_{\boldsymbol{\Theta}_j} (\nabla \cdot \boldsymbol{H}) d\boldsymbol{\Theta}_j =$$
$$= \int_{\partial\boldsymbol{\Theta}_j} \boldsymbol{H} \cdot d\boldsymbol{\tau} =$$
$$= \int_{\partial\boldsymbol{\Theta}_j} \boldsymbol{H} \cdot \boldsymbol{n} d\tau$$

where $\boldsymbol{n}$ is the outward unit vector normal to the acceptance boundary surface $\boldsymbol{\Theta}$, and $d\tau$ is the element of the set. The first equality highlights the notation used in the paper. We use just use one integral even if dealing with a multidimensional space since we indicate that we are integrating over a n-dimensional set. The second line contains the definition of the theorem. The third line takes into consideration that outward-pointing normals orient the surface (closed manifold $\partial\boldsymbol{\Theta}$). The change of variable is made to consider that the space of the n-surface is $1 - n$ dimensions, and the rotation makes it possible to write it in terms of the element scalar of the set. This definition is adapted from Weisstein (2002).

Moreover, to define $d\tau$ we take the definition of a general formula in Flanders (1973). In this definition, $d\boldsymbol{\tau}$ is the vectorial element on the boundary surface of $\partial\boldsymbol{\Theta}$, such that the resulting surface is oriented by outward pointing normals, so $d\boldsymbol{\tau} = \boldsymbol{n} d\tau$.

Derivation 2 (Unemployment value) *We start from the definition of the value of being unemployed:*

$$\begin{aligned}
V_u &= \frac{1}{1+r\Delta t}\left[-\bar{b}\Delta t + (1-\lambda\Delta t)V_u + \right. \\
&\quad \left. + \lambda\Delta t \mathbf{E}\max\left\{_e, V_u\right\}\right] + o(\Delta t)(1+r\Delta t) \\
V_u + r\Delta t V_u &= -\bar{b}\Delta t + V_u - \lambda\Delta t V_u + \\
&\quad + \lambda\Delta t \mathbf{E}\max\left\{V_e, V_u\right\} + o(\Delta t)(1+r\Delta t) \\
r\Delta t V_u &= -\bar{b}\Delta t - \lambda\Delta t V_u + \\
&\quad + \lambda\Delta t \mathbf{E}\max\{V_e, V_u\} + o(\Delta t)(1+r\Delta t)
\end{aligned}$$

We divide each side by Δt and we make $\Delta t \to 0$. Given the indeterminacy we apply L'Hôpital's rule so we can operate and introduce the value of being unemployed into the expectation.

$$\begin{aligned}
rV_u &= \frac{-\bar{b}\Delta t - \lambda\Delta t V_u + \lambda\Delta t \mathbf{E}\max\{V_e, V_u\} + o(\Delta t)(1+r\Delta t)}{\Delta t} \\
rV_u &= -\bar{b} + \lambda\mathbf{E}\max\{V_e - V_u, V_u - V_u\} \\
rV_u &= -\bar{b} + \lambda\mathbf{E}\max\{V_e - V_u, 0\} \\
rV_u &= -\bar{b} + \lambda\mathbf{E}_{\{\tilde{U}(\cdot)\geq 0|w_j;\boldsymbol{\theta}_i\}}\{V_e - V_u\}
\end{aligned}$$

Which is equal to equation 1 in the paper. Note that here the value of the expectation is with respect to the cases when $\{\tilde{U}(\cdot) \geq 0|w_j;\boldsymbol{\theta}\}$, that is when the value of the instantaneous utility is larger than or equal to the value of the outside option, conditional to the full information of firm types and their posted wages. The participation set then is defined by the set of all occupations in which the job seeker will accept to work given a proposed wage.

Derivation 3 (Employment value) *Taking the definition of the employment value in the text*

$$\begin{aligned}
V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) &= \frac{1}{1+r\Delta t}\left[(w_j\Delta t - c(\boldsymbol{r}_j,\boldsymbol{\theta})\Delta t) +\right. \\
&\left. +(1-\eta\Delta t)V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) + \eta\Delta t V_u\right] + o(\Delta t) \\
(1+r\Delta t)V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) &= w_j\Delta t - c(\boldsymbol{r}_j,\boldsymbol{\theta})\Delta t + \\
+(1-\eta\Delta t)V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) &+ \eta\Delta t V_u + o(\Delta t)(1+r\Delta t) \\
V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) + r\Delta t V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) &= w_j\Delta t - c(\boldsymbol{r}_j,\boldsymbol{\theta})\Delta t + V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) - \\
&- \eta\Delta t V_e(\boldsymbol{r}_j,\boldsymbol{\theta},w_j) + \eta\Delta t V_u + o(\Delta t)(1+r\Delta t)
\end{aligned}$$

We then group similar terms and divide by $(r+\eta)\Delta t$.

$$\begin{aligned}
(r+\eta)\Delta t V_e(\mathbf{r}_j,\boldsymbol{\theta},w_j) &= (w_j - c(\mathbf{r}_j,\boldsymbol{\theta}_i) + \eta V_u)\Delta t + \frac{o(\Delta t)(1+r\Delta t)}{(r+\eta)\Delta t} \\
V_e(\mathbf{r}_j,\boldsymbol{\theta}_i,w_j) &= \frac{(w_j - c(\mathbf{r}_j,\boldsymbol{\theta}) + \eta V_u)\Delta t}{(r+\eta)\Delta t} + \frac{o(\Delta t)(1+r\Delta t)}{(r+\eta)\Delta t}
\end{aligned}$$

Which, after simplification, and sending Δt to 0 is equal to equation 2 in the text.

$$V_e(\mathbf{r}_j,\boldsymbol{\theta},w_j) = \frac{w_j - c(\mathbf{r}_j,\boldsymbol{\theta}) + \eta V_u}{(r+\eta)}$$

Derivation 4 (Unfilled job value) *Starting from the definition in the text:*

$$\Upsilon_j = \frac{1}{1+r\Delta t}\Bigg[-k\Delta t + (1-q(\omega)\Delta t)\Upsilon_j + \\ + q(\omega)\Delta t \int \max\{J(\boldsymbol{r}_j,\boldsymbol{\theta}),\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta})\Bigg] + o(\Delta t)$$

$$\begin{aligned}
(1+r\Delta t)\Upsilon_j &= -k\Delta t + (1-q(\omega)\Delta t)\Upsilon_j + \\
&\quad + q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t) \\
\Upsilon_j + r\Delta t\Upsilon_j &= -k\Delta t + \Upsilon_j - q(\omega)\Delta t\Upsilon_j + \\
&\quad + q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t)(1+r\Delta t) \\
r\Delta t\Upsilon_j &= -k\Delta t - q(\omega)\Delta t\Upsilon_j + \\
&\quad + q(\omega)\Delta t \int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta}) + o(\Delta t)(1+r\Delta t)
\end{aligned}$$

Dividing by Δt and passing it to the limit we get that:

$$r\Upsilon_j = -k - q(\omega)\Upsilon_j + q(\omega)\int \max\{J(\mathbf{r}_j,\boldsymbol{\theta}),\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta})$$

$$\begin{aligned}
r\Upsilon_j &= -k + q(\omega)\int \max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j,\Upsilon_j-\Upsilon_j\}d\boldsymbol{F}(\boldsymbol{\theta}) \\
r\Upsilon_j &= -k + q(\omega)\int_{\boldsymbol{\Theta}_j} \max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j,0\}d\boldsymbol{F}(\boldsymbol{\theta}) \\
r\Upsilon_j &= -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j,\boldsymbol{\theta})-\Upsilon_j\}|\boldsymbol{\Theta}_J\right]
\end{aligned}$$

$$r\Upsilon_j = -k + q(\omega)N_j\mathbb{E}\left[\max\{J(\mathbf{r}_j, \boldsymbol{\theta}) - \Upsilon_j\}|\boldsymbol{\Theta}_j\right]$$

This last expression provides the value function of an unfilled vacancy, presented in equation 3 in the text. An equivalent definition without the conditional expectation is written below.

$$r\Upsilon_j = -k + q(\omega)\int\limits_{\boldsymbol{\Theta}_j}[J(\mathbf{r}_j, \boldsymbol{\theta}) - \Upsilon_j]f(\boldsymbol{\theta})d\boldsymbol{\theta}$$

Derivation 5 (Filled job value) *We start by the definition of the filled value of the vacancy presented in the text.*

$$J(\mathbf{r}_j, \boldsymbol{\theta}) = \frac{[m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j]\Delta t}{1 + r\Delta t} + \frac{(1 - \eta\Delta t)}{1 + r\Delta t}J(\mathbf{r}_j, \boldsymbol{\theta}) + \frac{\eta\Delta t}{1 + r\Delta t}\Upsilon_j + o(\Delta t)$$

We multiply both sides by $(1 + r\Delta t)$ and regroup all terms that have $J(\mathbf{r}_j, \boldsymbol{\theta})$ on the left hand side.

$$\begin{aligned}
(1 + r\Delta t)J(\mathbf{r}_j, \boldsymbol{\theta}) &= [m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j]\Delta t + (1 - \eta\Delta t)J(\mathbf{r}_j, \boldsymbol{\theta}) + \\
&+ \eta\Delta t\Upsilon_j + o(\Delta t)(1 + r\Delta t) \\
J(\mathbf{r}_j, \boldsymbol{\theta}_i) + r\Delta t J(\mathbf{r}_j, \boldsymbol{\theta}) &= [m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j]\Delta t + J(\mathbf{r}_j, \boldsymbol{\theta}) - \\
&- \eta\Delta t J(\mathbf{r}_j, \boldsymbol{\theta}) + \eta\Delta t\Upsilon_j + o(\Delta t)(1 + r\Delta t)
\end{aligned}$$

Grouping common terms and then solving for $J(\mathbf{r}_j, \boldsymbol{\theta})$, we have:

$$
\begin{aligned}
(\eta + r)\Delta t J(\mathbf{r}_j, \boldsymbol{\theta}) &= \{[m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j] + \eta \Upsilon_j\}(\Delta t) + o(\Delta t)(1 + r\Delta t) \\
J(\mathbf{r}_j, \boldsymbol{\theta}) &= \frac{[m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j] + \eta \Upsilon_j}{(\eta + r)} + \frac{o(\Delta t)(1 + r\Delta t)}{(\eta + r)\Delta t}
\end{aligned}
$$

After passing the limit Δt to 0, we get equation 4 in the text.

$$
J(\mathbf{r}_j, \boldsymbol{\theta}) = \frac{m_j(\boldsymbol{\theta}) - w_j + \eta(\Upsilon_j)}{(r + \eta)}
$$

Derivation 6 (Conditional Flow Vacancy) *Take equation 4 and replace the value of a filled job in equation 3. We get:*

$$
\begin{aligned}
r\Upsilon_j &= -k + q(\omega) N_j \mathbb{E}\left[\frac{[m(\boldsymbol{r}_j, \boldsymbol{\theta}) - w_j] + \eta \Upsilon_j}{(\eta + r)} - \Upsilon_j | \boldsymbol{\Theta}_j \right] \\
r\Upsilon_j &= -k + \frac{q(\omega) N_j}{(\eta + r)} \mathbb{E}\left[[m(\boldsymbol{r}_j, \boldsymbol{\theta}) - w_j] - r\Upsilon_j | \, \boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Equation 5 is the flow value of a vacancy given the posted wage w_j and it can be derived easily with a simple manipulation. Since the expected value can be operated linearly, and since the value of the vacant job does not depend on the participation set, we can write:

$$
\begin{aligned}
r\Upsilon_j &= -k + \frac{q(\omega) N_j}{(\eta + r)} \mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j] | \, \boldsymbol{\Theta}_j \right] - \\
&\quad - \frac{q(\omega) N_j r}{(\eta + r)} \Upsilon_j \\
r\Upsilon_j \left(\frac{(\eta + r) + N_j q(\omega)}{\eta + r} \right) &= -k + \frac{q(\omega) N_j}{(\eta + r)} \mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j] | \, \boldsymbol{\Theta}_j \right] \\
r\Upsilon_j \left((\eta + r) + N_j q(\omega) \right) &= -k(\eta + r) + \\
&\quad + q(\omega) N_j \mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j] | \, \boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Finally, solving for $r\Upsilon_j$ we arrive at equation 5 presented in the text.

$$r\Upsilon_j = \frac{-k(\eta + r) + q(\omega)N_j\mathbb{E}\left[[m(\mathbf{r}_j, \boldsymbol{\theta}_i) - w_j]|\,\boldsymbol{\Theta}_j\right]}{(\eta + r) + N_j q(\omega)}$$

Derivation 7 (Leibniz Rule) *Consider the function:*

$$G(w_j) = \int_{\boldsymbol{\Theta}:\tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} g(\boldsymbol{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

For the defined function $G(w_j)$, using the Leibniz multidimensional rule of differentiation under the integral leads to[10]:

$$\begin{aligned}\frac{dG(w_j)}{dw_j} = &\int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta})\right) d\boldsymbol{\theta} + \\ &+ \int_{\boldsymbol{\Theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0} \left(g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta})\right) \left(\nabla_{w_j} \boldsymbol{\theta} \cdot d\boldsymbol{\tau}\right)\end{aligned}$$

Here the gradient $\nabla_{w_j}\boldsymbol{\theta}$ is the velocity at which the boundary changes when changing w_j. In this definition, $d\boldsymbol{\tau}$ is the vectorial element on the boundary surface of $\partial\Theta$ such that $d\boldsymbol{\tau} = \boldsymbol{n} d\tau$, $\boldsymbol{n}$ is the outward unit vector normal to the acceptance boundary surface $\boldsymbol{\Theta}$, and $d\tau$ is the element of the set. Replacing the above equivalence, we get:

$$\begin{aligned}\frac{dG(w_j)}{dw_j} = &\int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} \left(\frac{\partial}{\partial w_j} g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta})\right) d\boldsymbol{\theta} + \\ &+ \int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) = 0} \big(g(\mathbf{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) (\nabla_{w_j} \boldsymbol{\theta} \cdot \boldsymbol{n})\big)\, d\tau\end{aligned}$$

Considering the outward velocity (divergence times the outward unit normal) of the boundary at each point, and with the definitions above we can write:

[10] In this definition we follow the appendix of (Veiga and Weyl, 2012, 2016), but we take the definition of a general space formula in Flanders (1973)

$$
\begin{aligned}
\nabla_{w_j}\boldsymbol{\theta}\cdot\boldsymbol{n} = \nabla_{w_j}\boldsymbol{\theta}\cdot \frac{\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\left\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\right\|} &= \\
&= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{\nabla_{\tilde{U}}\boldsymbol{\theta}\cdot\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\left\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\right\|} = \\
&= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\left\|\nabla_{\boldsymbol{\theta}}\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\right\|} = \\
&= \frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right\|}
\end{aligned}
$$

Replacing the result in the main equation leads us to equation A.1 in the paper.

$$
\begin{aligned}
\frac{dG(w_j)}{dw_j} = &\int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0}\left(\frac{\partial}{\partial w_j}g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\right)d\boldsymbol{\theta}+ \\
&+\int_{\boldsymbol{\theta}:\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})=0}\left(g(\mathbf{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}(\mathbf{r}_j,w_j,\bar{b};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right\|}\right)d\tau
\end{aligned}
$$

Derivation 8 (Optimal posted wage) *Using the definition of the conditional operator for the applicants pool in equation 3, we can rewrite it as:*

$$
kv = \frac{\int_{\boldsymbol{\theta}:\tilde{U}(\boldsymbol{r}_j,w_j,\bar{b};\boldsymbol{\theta})\geq 0} g(\boldsymbol{r}_j,w_j;\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}}{N_j}\lambda(v_j)\frac{N_j}{\eta+r}
$$

Using the result in eq. A.1

$$
\frac{\partial kv}{\partial w_j} = \int_{\boldsymbol{\Theta}_j} -f(\boldsymbol{\theta})d\boldsymbol{\theta} + \int_{\partial\boldsymbol{\Theta}}\left((m(\mathbf{r}_j,\boldsymbol{\theta})-w_j)\frac{\partial\tilde{U}}{\partial w_j}\frac{1}{\left\|\frac{\partial\tilde{U}}{\partial\boldsymbol{\theta}}\right\|}\right)d\tau = 0
$$

Using the definition of the conditional expected value we have that:

$$
\begin{aligned}
0 &= -N_j + M_j \frac{\int_{\partial\boldsymbol{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau}{\int_{\partial\boldsymbol{\Theta}_j} \frac{f(\boldsymbol{\theta}_j)}{\left\| \nabla_{\boldsymbol{\theta}_j} \tilde{U} \right\|} d\tau} \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} (m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \middle| \partial\boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} w_j \middle| \partial\boldsymbol{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Using the definition of the covariance we can rewrite $E[XY] = Cov[X, Y] + E[X]E[Y]$ for the second term and get:

$$
\begin{aligned}
0 = -N_j + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w_j}, m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] + \\
+ M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right] \mathbb{E}\left[m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial\boldsymbol{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial\boldsymbol{\Theta}_j \right]
\end{aligned}
$$

Derivation 9 (Optimal posted wage - Alternative) *Using the definition of the conditional operator for the applicants pool in equation 13, we can rewrite it as:*

$$
\Pi_j(\boldsymbol{r}_j, \boldsymbol{\theta}_i; w_j) = \max_{w} \frac{\int_{\boldsymbol{\theta}: \tilde{U}(\boldsymbol{r}_j, w_j, \bar{b}; \boldsymbol{\theta}) \geq 0} g(\boldsymbol{r}_j, w_j; \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta}}{N_j} N_j s(\boldsymbol{r}_j)
$$

Using the result in eq. A.1

$$
\frac{\partial \Pi_j}{\partial w_j} = \int_{\boldsymbol{\Theta}_j} -f(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\partial\boldsymbol{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau = 0
$$

Using the definition of the conditional expected value we have that:

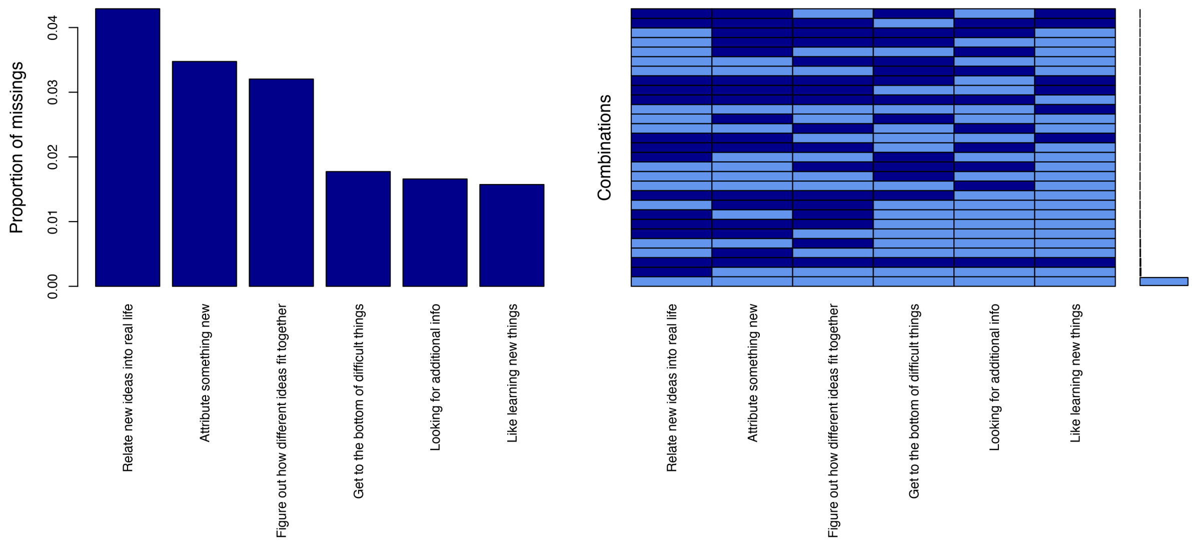
$$
\begin{aligned}
\frac{\partial \Pi_j}{\partial w_j} &= -N_j + M_j \frac{\int_{\partial \mathbf{\Theta}} \left((m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \frac{\partial \tilde{U}}{\partial w_j} \frac{1}{\left\| \frac{\partial \tilde{U}}{\partial \boldsymbol{\theta}} \right\|} \right) d\tau}{\int_{\partial \mathbf{\Theta}_j} \frac{f(\boldsymbol{\theta}_j)}{\left\| \nabla_{\boldsymbol{\theta}_j} \tilde{U} \right\|} d\tau} \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} (m(\mathbf{r}_j, \boldsymbol{\theta}) - w_j) \middle| \partial \mathbf{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \mathbf{\Theta}_j \right] - M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} w_j \middle| \partial \mathbf{\Theta}_j \right] \\
&= -N_j + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \mathbf{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \mathbf{\Theta}_j \right]
\end{aligned}
$$

Using the definition of the covariance we can rewrite $E[XY] = Cov[X, Y] + E[X]E[Y]$ for the second term and get:

$$
\begin{aligned}
\frac{\partial \Pi_j}{\partial w_j} &= -N_j + M_j Cov\left[\frac{\partial \tilde{U}}{\partial w_j}, m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \mathbf{\Theta}_j \right] + \\
&\quad + M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \mathbf{\Theta}_j \right] \mathbb{E}\left[m(\mathbf{r}_j, \boldsymbol{\theta}) \middle| \partial \mathbf{\Theta}_j \right] - w_j M_j \mathbb{E}\left[\frac{\partial \tilde{U}}{\partial w_j} \middle| \partial \mathbf{\Theta}_j \right] = 0
\end{aligned}
$$

C Appendix III

Figure A.1: Patterns of missingness for Non Cognitive questions



Source: PIAAC France 2012

Dimension	Variable	Weight
Plausible value - Numeric	PVNUM1	0.763
Plausible Value - Literacy	PVLIT1	0.646

	Variable	Factor1
Relate new ideas into real life	I_Q04b	0.581
Like learning new things	I_Q04d	0.681
Attribute something new	I_Q04h	0.485
Get to the bottom of difficult things	I_Q04j	0.723
Figure out how different ideas fit together	I_Q04l	0.728
Looking for additional info	I_Q04m	0.612

Table A.1: Weighted and Unweighted moments - France

	$Moment_w$	$Moment_u$
Cognitive skill - Q_{10}	0.382	0.380
Cognitive skill - Q_{25}	0.483	0.488
Cognitive skill - Q_{50}	0.583	0.586
Cognitive skill - Q_{75}	0.669	0.673
Cognitive skill - Q_{90}	0.738	0.743
Non-cognitive skill - Q_{10}	0.477	0.485
Non-cognitive skill - Q_{25}	0.592	0.596
Non-cognitive skill - Q_{50}	0.685	0.685
Non-cognitive skill - Q_{75}	0.775	0.779
Non-cognitive skill - Q_{90}	0.881	0.881
Cognitive Requirement - Q_{10}	0.096	0.096
Cognitive Requirement - Q_{25}	0.138	0.138
Cognitive Requirement - Q_{50}	0.280	0.280
Cognitive Requirement - Q_{75}	0.415	0.415
Cognitive Requirement - Q_{90}	0.558	0.558
Non - Cognitive Requirement - Q_{10}	0.156	0.205
Non - Cognitive Requirement - Q_{25}	0.328	0.328
Non - Cognitive Requirement - Q_{50}	0.612	0.634
Non - Cognitive Requirement - Q_{75}	0.700	0.732
Non - Cognitive Requirement - Q_{90}	0.947	0.947
Hourly wages - Q_{10}	7.910	7.972
Hourly wages - Q_{20}	9.891	9.971
Hourly wages - Q_{30}	10.889	10.944
Hourly wages - Q_{40}	11.310	11.455
Hourly wages - Q_{50}	11.857	12.171
Hourly wages - Q_{60}	12.689	13.117
Hourly wages - Q_{70}	15.807	16.315
Hourly wages - Q_{80}	17.544	17.712
Hourly wages - Q_{90}	21.481	22.478
Mean Employment	0.861	0.881