The Great Equalizer: How Firms Reduce Wage Inequality*

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Abstract

We study how firms impact the dynamics of earnings inequality. Using linked employer-employee-job title data from Portugal, we show that the compression of firm pay premiums accounted for 86 percent of the large decline in wage inequality over the past decades despite an increase in worker heterogeneity. A decline in job title heterogeneity and a decreasing concentration of high-paying jobs in high-quality firms account for the remainder of this decline. The compression in firm pay premiums resulted from two factors: (i) a decrease in the pay gap between otherwise similar workers working in more or less productive firms, and (ii) declining returns to working with highly skilled co-workers. Using a novel mediation analysis, we show that the effects of firm productivity and skill composition on wage inequality are broader than previously estimated using conventional AKM variance decompositions. While previous studies assume that firm pay policies fully mediate the relationship between firm characteristics and wage inequality, we show that 76 percent of firm productivity affects wage inequality through factors other than firm pay premiums, such as worker quality and job title pay premiums.

Keywords: Labor Markets, Wage Inequality, Firms, Institutions, Concentration
JEL Classification: J00, J31, J40

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1 Introduction

Wage inequality has taken center stage in the policy debate in developed and developing countries. Significant resources and a tremendous amount of effort have been devoted to fighting rising inequality. Despite the relevance of firm characteristics for wage inequality (Card et al., 2013; Alvarez et al., 2018; Song et al., 2019; Bonhomme et al., 2019; Messina and Silva, 2021), policies focusing on firms are scarce. Behind this phenomenon lies a lack of understanding of why and how firms matter for wage inequality dynamics. Which firm characteristics matter and what is the mechanism whereby these characteristics impact the dynamics of wage dispersion? Answering these questions is critical to help better calibrate inequality-mitigating policies and ultimately contribute to a more efficient allocation of resources.

This paper exploits a large reduction in Portuguese wage inequality to provide an answer to these questions. Figure 1 shows that the decline in Portuguese wage inequality over the twenty-first century (a decline of around 20 percent) was almost entirely driven by declining dispersion in the average wage paid by firms to their employees[1] (*between-firm inequality*)[2]. We show that the decline in between-firm inequality was driven by a compression in the wage premium to firms' intrinsic characteristics (*firm pay premiums*), rather than by changes in the extent to which highly productive workers and highly productive firms matched with each other. The compression of wage premiums to occupations' intrinsic features (*job title pay premiums*) and changes in the extent to which good occupations tend be located in highly productive firms were also important for this decline in wage inequality, albeit less so.

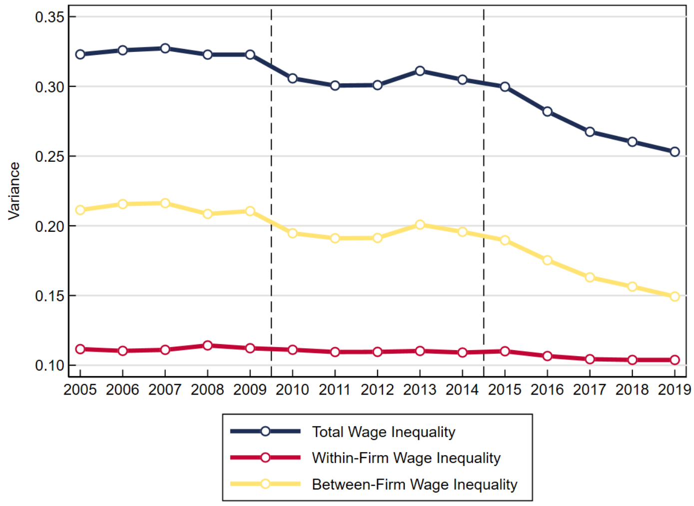
We provide the first comprehensive analysis of the direction and magnitude of changes in firm size, labor productivity, firm skill requirements (*i.e.* the level of skill intensity of jobs within a firm), firm exposure to the minimum wage, and market concentration on firm pay premium dispersion and job title pay premium dispersion. Moreover, guided by a search and matching model, we investigate whether the observed decline in earnings inequality was driven by declining returns to firm characteristics (henceforth, *passthrough effect*), or by firms becoming more homogeneous in their characteristics over time (henceforth, *composition effect*). For each characteristic, we show that a weakening passthrough from firm characteristics to firm pay overcompensates changes in the distribution of firm characteristics and is entirely responsible for the decrease in earnings inequality observed in Portugal. Taken together, declining returns to labor productivity and to working with highly skilled coworkers were the main drivers behind the compression of firm pay premiums.

We complement the commonly used workhorse framework linking firm characteristics and wage inequality in two directions. First, when explaining wage inequality dynamics

[1] Overall inequality dynamics may stem from systematic differences in pay across firms (*between-firm inequality*) or from differences in pay within each firm (*within-firm inequality*) (Card et al., 2013; Alvarez et al., 2018; Song et al., 2019; Messina and Silva, 2021).

[2] In a nonparametric density counterfactual decomposition following Autor et al. (2005), Machado and Mata (2005) and Song et al. (2019), we show a counterfactual evolution of inequality had it not been for between-firm inequality. We show that overall inequality would have stagnated or increased slightly had it not been for the decline in between-firm inequality. We present our results for this decomposition in panel (c) Figure A1, in Appendix A.

Figure 1: Portuguese Wage Inequality Dynamics (2005-19).



Source: *Quadros de Pessoal*, $2005-19$.
Note: This figure depicts the yearly evolution of the variance of hourly wages ("total wage inequality") over 2005-19, decomposed in within-firm inequality and between-firm inequality components. The vertical sum of the two components adds up to overall inequality for each year. Firm variance is computed based on average log earnings and is weighted by the number of workers in the firm. Within-firm variance is based on the difference between a worker's log hourly earnings and the average wage paid by his or her firm. Additional details on how to implement this estimation are provided in Appendix B.

via the Abowd, Kramarz and Margolis (1999) (AKM) variance decompositions, previous studies focused on how changes in firm characteristics affect wages and their dispersion through changes in time-invariant firm quality. By doing so, these studies implicitly neglect the possibility that changes in firm characteristics can also affect wages and their dispersion directly, independently of time-invariant firm quality. There is the risk that the link between changes in firm characteristics and changes in wage inequality could become irrelevant once the analysis accounts for the direct effect of firm characteristics on wages. To address this concern, we propose a form of mediation analysis in the vein of Gelbach (2016), to show how firm characteristics can affect wages and wage dispersion both indirectly (via firm fixed effects) and directly (via omitted variable bias). Our findings support the link between firm characteristics and wage inequality mediated by firm fixed effects: a substantial fraction of changes in firm characteristics affects wages and their dispersion via firm fixed effects. However, the inclusion of the direct link is not innocuous: part of the change in firm characteristics affects wages and wage inequality directly. Only about 24 percent of the effect of labor productivity and 20 percent of the effect of skill composition affect wage dispersion via the indirect effect. Put differently, workplace policies on worker pay contribute to 24 percent of the effect of labor productivity and 20 percent of the effect of skill composition on wage dispersion, after controlling for workers' characteristics and job titles.

Second, recent literature has raised concerns about the variance of the firm component in the AKM model. Alternative models have emerged (Bonhomme et al., 2019), and various corrections have been posited to account for autocorrelation (Andrews et al., 2008) or control for the network structure of the data (Kline et al., 2020). In this study, we overcome these limitations by incorporating into our AKM model detailed occupational information that accounts for differences in job tasks and institutional factors. When we include this information – and adjust the network structure by removing non-articulation points – we demonstrate the real importance of the variance of the firm pay premium component for wage inequality.

With these concerns in mind, we investigate the drivers of between-firm inequality through a two-step procedure. In the first step, we disentangle observed wage dynamics into the contributions of worker, firm, and job title heterogeneity, as well as their co-movement. Following Card et al. (2013), we provide evidence that the strong separability and exogenous mobility AKM assumptions are met in the data. In our first step, we initially do not control for worker and firm observable characteristics. Rather, in the second step, we explain firm fixed effects based on observable characteristics. We verify empirically that this two-step procedure is free of omitted variable bias. That is, in the second step, we inquire how observed firm characteristics translate into (i) the expected value of firm pay premiums, and (ii) the dispersion of these premiums. To evaluate how firm characteristics translate into firm fixed-effect dispersion, we project the nonparametric variance counterpart, namely, the recentered influence function (RIF) (Firpo et al., 2009, 2018) into these covariates. We employ the RIF function as it is a computationally simple regression framework that evaluates how a set of covariates affects the second moment of the firm pay premium distribution.[3]

From our first step, we find that heterogeneity across workers (related to their fixed characteristics) is the strongest driver of wage variance in levels, explaining between 44 and 52 percent of overall wage inequality. Yet, firms also play a key role in the level of wage inequality, with firm heterogeneity explaining between 19 and 22 percent of total wage variance. The association between high-earning workers and high-paying jobs accounts for between 8 and 11 percent of total wage dispersion. These results are consistent with the literature (see, for example, Portugal et al. (2018) for evidence for Portugal, and Alvarez et al. (2018) and Card et al. (2013) for evidence for Brazil and Germany, respectively). Overall, we find that the reduction in wage dispersion was due to a reduction in firm fixed-effect dispersion (around 60 percent), a decline in job title fixed effect dispersion (around 10 percent), and a decline in *firm-job assortativity* dispersion (around 7 percent).[4] Meanwhile, stagnation of the dispersion of worker fixed effects prevented a more substantial decline in overall inequality.

From our second-step analysis of covariate projection onto firm fixed effects, we find that firm observables (firm size, market concentration, labor productivity, and the share of workers earning the minimum wage) explain around one-third of the variability of firm fixed effects. We find that labor market concentration and the share of workers earning the minimum wage contribute negatively to the expected value of firm fixed effects, while firm skill requirements, labor productivity, and market concentration contribute positively, which is consistent with our conceptual framework. We also find

[3]There are two technical reasons why the RIF framework is more appropriate than a conditional quantile regression framework. First, a quantile regression framework would not allow directly identifying the determinants of the second moment of the fixed effects distribution. Second, if there is no full support of the chosen regressors, we can not include those covariates in the quantile regression.

[4]We understand *firm-job assortativity* as the tendency in the economy for better types of jobs to be present in firms in which employers offer higher wages.

that firm skill requirements contribute positively to dispersion of the firm fixed effects, along with the share of minimum wage workers and product market concentration. Our Oaxaca-Blinder (Oaxaca, 1973; Blinder, 1973; Kitagawa, 1955) decomposition reveals that the decline in dispersion explained by these characteristics was driven by the reduction in the premiums for the characteristics rather than due to changes in the distribution of these characteristics. We show that had the returns to these characteristics not declined, the dispersion of firm fixed effects would have increased by 8 percent, which would have contributed to a 1.5 percent increase in wage inequality. We conclude that lower passthrough from firm characteristics to pay played a key role in compressing firm pay premiums. A key finding of our endeavor is that firm skill requirements and labor productivity were the main drivers of the compression of the firm pay premium dispersion. The decrease in the passthrough of labor productivity and firm skill requirements, taken together, explain the largest share of the decrease in the variance of the firm fixed effects.

Our findings have important implications for policymakers interested in mitigating inequality. As educational attainment levels have continued to rise steadily in developed economies, the marginal impact of educational policy investment has decreased, driven by diminishing returns. Our findings suggest that policies that limit product market concentration or propel idiosyncratic technology adoption for technologically laggard firms may be effective in tackling inequality.

Related Literature We contribute to three strands of the literature. First, we contribute to the literature investigating the role of firms in wage inequality dynamics. Studies such as Dunne et al. (2004), Faggio et al. (2010), Helpman et al. (2017), Sorkin (2017), Mueller et al. (2017), and Håkanson et al. (2021) document that firm characteristics matter for wage dispersion. Following Card et al. (2013), a number of studies have disentangled the sorting and firm pay premium components of between-firm wage inequality (Barth et al., 2016; Alvarez et al., 2018; Song et al., 2019; Messina and Silva, 2021). Card et al. (2013) rely on the AKM framework in their study of Germany and find that increasing pay premiums and assortativity explain a large share of the growth in wage inequality (60 percent). Alvarez et al. (2018) find that the decreasing wage inequality in Brazil can be explained by the decrease in the variance of the pay premium (40 percent) and the decrease in assortativity (23 percent). Messina and Silva (2021) report similar findings for Latin American countries (Ecuador, Brazil, and Costa Rica). Barth et al. (2016) and Song et al. (2019) find that a large share of the rise in inequality in the United States is due to a rise in between-firm inequality, with sorting being the key underlying driver. Among these studies, Alvarez et al. (2018) establish a link between value added and the level of firm pay premiums. They find that more productive firms pay significantly more even after controlling for sorting. Bloom et al. (2018) document a negative relationship between firm size and the pay premium. Yet, these papers remain silent on the contribution of these characteristics to the *dispersion* of firm pay premiums. We contribute to this literature by examining the institutional and firm-level channels driving changes in firm and job title pay premium dispersion. To the best of our knowledge, we are the first to use the RIF to this end.

Second, we contribute to the recent literature highlighting the importance of job title heterogeneity for wage dispersion. Job titles capture the institutional and task compensation heterogeneity of the roles and occupations inside the firm (Cardoso et al., 2016; Portugal et al., 2018; Raposo et al., 2021). For example, Dustmann et al. (2009) provide evidence that different occupations are not distributed uniformly in the support of the wage distribution. Consequently, occupation types can drive different wage inequality dynamics. A similar finding for routine and non-routine tasks (cognitive or manual) explains the impact of job polarization on the distribution of wages (Goos and Manning, 2007). Another strand of the recent literature highlights that including a richer description of the types of jobs (tasks and skills) could shed light on the dynamics of wage inequality (Autor and Handel, 2013; Acemoglu and Restrepo, 2019). Yet, none of these papers explores the relationship between firm characteristics and job title pay premium dispersion and their impact on earnings inequality.

Disregarding job titles while studying earnings inequality dynamics may not only compromise the robustness of our inferences, but also jeopardize our ability to determine the exact drivers behind changes in the wage distribution. Job titles matter for robust inference for several reasons. First, if they are important for wage setting, their omission might jeopardize estimation of the firm and worker fixed-effect components through omitted variable bias. Second, job titles matter since firm pay premiums and worker-specific characteristics may be correlated with job title fixed effects, and these covariances might thus matter for inequality. A third danger of omitting job titles is that if the error term is heteroskedastic (Andrews et al., 2008), the estimated variance explained by the pay premium and assortativity might be biased (Kline et al., 2020) (henceforth, KSS), also thereby affecting the ability to carry out inference. We circumvent these obstacles by focusing on the unique context of Portugal, where rich administrative linked employer-employee job title data are available.

Third, our study relates to the literature on robust identification and inference of models with high-dimensional fixed effects (Andrews et al., 2008; Kline et al., 2020). A robust estimation could reveal a reduction in wage dispersion and the importance of the firm pay premium component (Bonhomme et al., 2019; Kline et al., 2020). In this paper, we extend the KSS correction to include worker, firm, and job title fixed effects. We show that incorporating a third high-dimensional fixed effect improves the connectivity of the network and assures robust estimation. As shown in previous studies, the KSS correction considerably reduces the sample size, potentially reducing wage variance and the contribution of the firm component to overall variance (Bonhomme et al., 2019; Kline et al., 2020). In this setting, we extend the KSS leave-one-out methodology to the three high-dimensional fixed effects setting. By considering job titles, the loss in sample size when removing the no articulation vertex (leave-one-out) sample is mitigated. This contributes to the robust estimation of firm pay premium dispersion, worker fixed effect dispersion, and job title fixed effect dispersion.

Outline The remainder of the paper is organized as follows. Section 2 lays out the conceptual framework as a motivation for our empirical analysis. Section 3 describes our main data sources and presents descriptive statistics. Section 4 presents a set of stylized facts that suggest that firms and job titles are major components behind the fall in wage inequality. Section 5 explores the drivers of between-firm inequality dynamics. Section 6 presents our mediation analysis. Section 7 addresses robustness and threats to identification. Section 8 concludes.

2 Conceptual Framework

This section outlines the conceptual framework that guides our empirical approach. We rely on the canonical AKM two-way fixed effects model (Abowd et al., 1999), in which the log wage is additive on worker, firm, and job title fixed effects. Like Alvarez et al. (2018), we estimate a *restricted* wage model, of the following form:

$$w_{it} = \alpha_i + \psi_{j(i,t)} + \phi_{k(i,t)} + \tau_t + \epsilon_{it} \tag{1}$$

where α_i is a worker effect, which captures the time-invariant unobserved characteristics of each worker; $\psi_{j(i,t)}$ is a firm effect, which captures the firm pay premium component; $\phi_{k(i,t)}$ is a job title fixed effect, which captures the time invariant unobserved characteristics of the different occupations; τ_t is a time fixed effect; and ϵ_{it} is an error term component. The error component is assumed to follow a conditional mean-zero assumption (see equation 2), ruling out worker mobility because of the error component. We follow Card et al. (2013) to test empirically whether that is the case in the data, and that the gains of switching workers across quartiles of the distribution are symmetric.[5]

$$\mathbb{E}[\epsilon_{it}|\alpha_i, \psi_{j(i,t)}, \phi_{k(i,t)}, \tau_t] = 0 \tag{2}$$

The model is said to be *restricted* because it does not incorporate observable time-varying characteristics. Our aim is to understand the determinants of the nonvarying components, following Alvarez et al. (2018), Song et al. (2019), and Messina and Silva (2021). In a later stage, we project the estimated fixed effects into time-varying characteristics. The selection of covariates employed in our analysis is guided by a combination of recent empirical literature, a set of stylized facts that will be presented later in the paper, and the theoretical wage-setting model employed.[6] Specifically, the model posits that total factor productivity, bargaining power, labor cost frictions, and the firms' capital stock per worker can be mapped into the firm pay premium. Although we use direct measures for each of these factors, the resulting structure of the firm pay premium allows us to decompose the firm pay premium dispersion into changes in the distribution of firm characteristics (*composition*) and changes in the compensation for those factors (*passthrough*). Following our model results, the firm pay premium is defined as directly related to four components.

$$\psi_{j(i,t)} = \beta_t \frac{z_j f(k_j)}{(1+\gamma_j)} \tag{3}$$

First, it depends on total factor productivity. Second, it is also a function of the worker bargaining weight, β_t, which is assumed to be constant across workers and firms but floats freely over time. Third, the firm-specific component also depends on a labor cost friction. The higher is the friction, γ_j, the lower is the firm-specific component, $\psi_{j(i,t)}$. Finally, the firm-specific component depends on the capital stock per worker. The capital stock is a proxy for the firm-specific skill requirements in production. A firm that invests

[5] The detailed empirical test of the exogenous mobility assumption is presented in section 7.

[6] The model builds on insights from a simple Diamond-Mortensen-Pissarides model (Mortensen and Pissarides, 1994; Pissarides, 1984a,b, 1985), with two-sided worker-firm heterogeneity (Postel–Vinay and Robin, 2002; Cahuc et al., 2006). We provide a detailed derivation of our model in the Online Supplemental Appendix, while the main text presents its key insights. This setting pins down the role of firms as key drivers of wage inequality dynamics.

a large amount of capital will require different types of workers for the different tasks and the contents of the jobs. After estimating the reduced AKM specification (equation 1), we decompose the wage variance into the worker, firm, and job title components. In doing so, we explicitly evaluate the contribution of firms' heterogeneity to wage inequality (captured by the variance in wages), as well as the heterogeneity of workers and job titles. Concretely, for each subperiod, the wage variance is linearly decomposed as:

$$Var(w_{it}) = Var(\alpha_i) + Var(\psi_{j(i,t)}) + Var(\phi_{k(i,t)}) + Var(\tau_t) + Var(\epsilon_{it}) + \mathbf{2C}^T \tag{4}$$

where $Var(w_{it})$ stands for the variance of log real hourly wages. $Var(\alpha_i)$, $Var(\psi_{j(i,t)})$, $Var(\phi_{k(i,t)})$, and $Var(\tau_t)$ are the variances of worker, firm, job title, and year fixed effects, respectively. These variances represent, respectively, wage heterogeneity across workers related to their fixed characteristics, wage heterogeneity across firms from their distinct pay premiums, and wage heterogeneity across different job titles. The term $Var(\epsilon_{it})$ stands for the variance of the error term. The term $\mathbf{2C}^T$ is a 1 by 6 vector including the covariance between all combinations of the terms on the right-hand side of equation 1.

In levels, inequality in wages stems from inequality across firms in technology, capital intensiveness, and labor cost frictions. It also stems from heterogeneity in worker characteristics, causing divergence in their outside options. Finally, inequality also depends on the sorting of workers between firms and job titles. Which of these effects makes a stronger contribution to overall wage inequality in period t depends on the relative bargaining power of the agents. If workers have full bargaining power, then $\beta_t = 1$ so that dispersion in firm heterogeneity plays a large role in explaining the level of wage inequality. How the variance of firm-specific characteristics translates into wage inequality depends on both a composition and the passthrough component.

$$Var(\psi_{j(i,t)}) = \underbrace{(\beta_t)^2}_{Passtrhough} \times \underbrace{Var\left(\frac{z_j f(k_j)}{1+\gamma_j}\right)}_{Composition}$$

The change in inequality over time can be assessed by evaluating the change in the variance of wages across consecutive periods (by taking first differences in equation 4). In dynamics, as long as there are changes in bargaining power over time, there will be changes in inequality over time. Concretely, if the bargaining power of workers increases over two consecutive time periods, so that $\Delta\beta_t^2 > 0$, then a smaller dispersion of firm characteristics leads to less inequality.

3 Data and Descriptive Statistics

This paper's empirical analysis draws on two main datasets.

Quadros de Pessoal (QP) First, we use *Quadros de Pessoal* (QP), a matched employer-employee data set collected by the Portuguese Ministry of Employment. QP covers and follows over time all Portuguese private sector workers and firms with more than one worker, having close to 300,000 firms[7] and more than 2.5 million

[7]Since most of our analysis deals with firms, it is useful to note that a firm in the Portuguese microdata is defined as an entity conducting an "economic activity" and having a registered office, based on a fiscal

worker observations each year over 2005-19. We restrict the analysis to full-time dependent workers between ages 18 and 65 years (working-age population). The data set provides comprehensive information on workers' demographic characteristics (age, gender, schooling, and so forth) and job characteristics (occupational group, professional category, wage,[8] hours worked, firm tenure, and so forth). For each worker, the employing firm is uniquely identified through a firm identification code. The firm-level characteristics in QP include, among others, sales, number of employees, equity, percentage of foreign capital, geographical location and date of creation, and the industry code according to the Portuguese Classification of Economic Activities (*Classificação Portuguesa das Atividades Económicas* (CAE)).[9] We provide additional details on these data and variables in the Online Appendix. These data have been used recently by Carneiro et al. (2012), Card et al. (2016), Card and Cardoso (2021), Raposo et al. (2021), and Carneiro et al. (2022).

Table 1 displays the descriptive statistics for selected variables and indicators, by subperiods and for the overall period. The sample contains data on individual workers for which a fixed effect was estimated. The table presents statistics for both the largest connected set (Abowd et al., 2002)[10], and the *leave-one-out sample* (Kline et al., 2020). Our definition of the leave-one-out sample extends the KSS methodology to include worker, firm, and job title fixed effects. The three-way leave-one-out sample is constructed under a simple assumption on the network structure: workers are connected to firms, and firms are connected to job titles. We rule out the less restrictive possibility that workers connect directly to both firms and job titles, since that would decrease the probability of finding articulation points in the network. A first interesting result from Table 1 is that the *three-way leave-one-out sample* is similar in size to the largest connected set. This result is in contrast to previous evidence, in which a two-way leave-one-out sample results in a large decrease in sample size, changes the estimated variance components, and affects inference (Bonhomme et al., 2019; Kline et al., 2020). Extending to three components allows us to preserve workers who did not move across firms but have similar job titles, sharing the institutional framework and job conditions that keep them connected to the network.

Sistema de Contas Integradas das Empresas (SCIE) We also use the *Sistema de Contas Integradas das Empresas* (SCIE), which is a longitudinal, firm-level data set collected by Statistics Portugal. This dataset links with QP through the unique firm

identifier. We focus on firms rather than establishments since there are reasons to believe that factors giving rise to employer-specific pay operate at the firm level, rather than the establishment level (such as corporate culture or management practices).

[8]We use real log hourly total earnings as our measure of labor income (this includes base labor earnings for normal hours worked, plus regular payments and premiums). As in Alvarez et al. (2018), this measure does not contain other sources of earnings such as capital gains or in-kind transfers. To benchmark our data and show its quality, in Figure A4, in Appendix A, we show that this distribution of earnings in QP tracks closely that of the survey data *Inquérito às Condições de Vida e Rendimento* (ICOR).

[9]Given that the Portuguese classification of firms' economic sectors (CAE) was revised in 2007 to match the Nomenclature of Economic Activities (NACE) Revision 2, a concordance was put together to ensure harmonization with the sector classifications from previous years.

[10]The largest connected set gives the largest sample in which all firms and job titles are connected by worker mobility. In support of the mobility assumption, we present the mobility across firms and job titles in Table A1, in Appendix A.

Table 1: Summary Statistics

	2005-09	2010-14	2015-19	2005-19
Largest connected set				
Observations	8,161,585	7,668,852	7,881,089	26,516,202
Firms	294,550	247,627	232,870	481,992
Workers	2,254,434	2,061,354	2,145,383	3,684,974
Job titles	34,596	52,696	43,247	82,015
Movers across firms	398,716	297,777	392,864	1,495,186
Movers across job titles	931,510	1,405,720	1,015,331	2,798,628
Mean $\log(w)$	1.7092	1.7395	1.7827	1.7328
Variance $\log(w)$	0.3280	0.3084	0.2764	0.3049
Leave-one-out sample (KSS)				
Observations	8,146,049	7,655,728	7,864,707	26,502,639
Firms	293,682	246,907	232,304	481,274
Workers	2,241,581	2,051,065	2,135,020	3,672,511
Job titles	34,344	52,275	42,850	81,913
Movers across firms	398,288	297,410	392,589	1,494,914
Movers across job titles	931,059	1,405,275	1,014,265	2,798,429
Mean $\log(w)$	1.7095	1.7396	1.7823	1.7329
Variance $\log(w)$	0.3280	0.3084	0.2764	0.3049

Source: *Quadros de Pessoal*, $2005-19$.
Note: The table displays descriptive statistics on the number of movers and units. The top part of the table presents statistics for the largest connected set (three-way), which gives the largest sample in which all firms and job titles are connected by worker mobility. The bottom part of the table displays statistics for the *three-way leave out sample*, which is the largest connected set such that every firm and job title remains connected after removing any single worker from the sample. The first three columns present the summary information for each subperiod, and the last columns present the key descriptive statistics for the whole sample over 2005-19.

identification code. SCIE covers all firms (companies, individual entrepreneurs, and self-employed) that produce goods or services during the year, excluding firms in the insurance and financial sector, those that produce agricultural products, and entities that are not market oriented. From 2005 to 2019, each year has more than 1 million firm observations detailing their economic activity (for example, CAE industry code, geographical location (according to the Nomenclature of Territorial Units for Statistical Purpose (NUTS II)), birth/death, and number of workers) and accounting statements. Generically, the dataset includes information on financing and accounting variables. Employment and labor productivity variables can also be extracted from SCIE.

4 Evolution of Wage Inequality in Portugal

4.1 Stylized Facts

Wage inequality in Portugal has declined continuously over the course of the twenty-first century, by a staggering 20 percent. This is shown by three different metrics (Gini coefficient, log 90/10 percentile ratio, and variance of log wages) in panel (a) in Figure A1, in Appendix A. Moreover, this decline in inequality was characterized by four empirical facts.[11] First, the decrease in inequality affected the overall support of the income distribution, but the decrease was larger in the lower end of the distribution (see panel (b) in Figures A1 and A10). Second, when we decompose wage dispersion into its within-firm and between-firm components (see Figures 1 and A1), the latter is the most important in levels (around 60 percent). Moreover, the decline in the between-firm component decline over 2015-19 accounts for the bulk of the decline in wage inequality over that time span.[12] Third, when we decompose the variance of wages into the within-skill and between-skill components, the within-skill component is the largest one in levels (see Figure A11). Even if both components decreased, the change in the between-skill component is larger and thus drove the fall in overall wage variance. This highlights the relevance of having a variable that captures different skills across demographic groups, such as job titles.[13] Fourth, historically, large firms have paid significantly higher wages. When we analyze the evolution of this relationship over time, we find that the relationship between firm size and wages in Portugal has weakened considerably, consistent with the findings of Bloom et al. (2018) (see Figure A12).

4.2 Contribution of Workers, Firms and Job Titles to Inequality

The model presented in equation 1 and its variance decomposition in equation 4 show that changes in wage inequality could be driven by worker heterogeneity, firm heterogeneity, job title heterogeneity, or sorting. In this section, we investigate which component drove the decline in wage inequality in Portugal. Our stylized facts provide preliminary evidence that firms and occupations are important in shaping wage inequality dynamics in Portugal. However, the fact that different firms may have very different wage profiles due to systematic differences in the workers they hire or type of jobs (Gerard et al., 2021), together with the fact that similar skills might be rewarded differently across firms, implies that the initial within-between mechanical decompositions conducted above do not capture these effects jointly.

To assess the contributions of worker, firm, and job title heterogeneity to wage inequality, we start by estimating equation 1 in three sub-periods: 2005-09, 2010-14 and 2015-19[14]. Then, we decompose the variance of wages according to equation 4.

Table 2 presents the AKM variance decomposition for Portugal for the three

[11] Appendix B provides a detailed explanation on the unambiguous decline of labor income inequality in Portugal and a detailed description of the stylized facts on earnings inequality in the country.

[12] This pattern is also present across sectors (see Figure A8, in Appendix A), across firm sizes (see Figure A7), and within several demographic groups (see Table A2, in Appendix A).

[13] This finding stands in contrast to what is found in the United States, where the dispersion in wages occurred within skills, resulting in an increase in inequality (Autor et al., 2020).

[14] We have verified that our results are robust to selecting larger subperiods and overlapping periods. We present these results in Tables A9, A10 and A11, in Appendix A

Table 2: Wage variance decomposition - AKM

	2005-09		2010-14		2015-19		2005-19	
	Value	Share (%)	Value	Share (%)	Value	Share (%)	Value	Share (%)
Variance $\log(w)$								
Plug In	0.3280		0.3084		0.2764		0.3049	
Leave out (KSS)	0.3279		0.3083		0.2761		0.3049	
Variance workers effects								
Plug in	0.1443	44.01	0.1590	51.55	0.1330	48.11	0.0989	32.45
Leave out (KSS)	0.1444	44.02	0.1590	51.58	0.1330	48.17	0.0989	32.44
Variance firm effects								
Plug in	0.0820	24.99	0.0673	21.82	0.0519	18.78	0.0589	19.32
Leave out (KSS)	0.0819	24.99	0.0673	21.82	0.0519	18.79	0.0589	19.32
Variance job title effects								
Plug in	0.0190	5.81	0.0163	5.30	0.0136	4.91	0.0212	6.97
Leave out (KSS)	0.0190	5.80	0.0163	5.29	0.0135	4.89	0.0213	6.97
Covariance of worker-firm ($2\times$)								
Plug in	0.0163	4.98	0.0087	2.83	0.0206	7.44	0.0342	11.22
Leave out (KSS)	0.0164	4.99	0.0087	2.83	0.0205	7.42	0.0342	11.22
Covariance of worker-job title ($2\times$)								
Plug in	0.0336	10.24	0.0258	8.38	0.0300	10.85	0.0441	14.47
Leave out (KSS)	0.0336	10.23	0.0258	8.36	0.0299	10.83	0.0441	14.47
Covariance of firm-job title($2\times$)								
Plug in	0.0125	3.81	0.0149	4.84	0.0103	3.73	0.0177	5.81
Leave out (KSS)	0.0125	3.80	0.0149	4.83	0.0102	3.69	0.0177	5.81
Coefficient of determination R^2								
Plug in	0.9137		0.9254		0.9161			0.8892
Leave out (KSS)	0.9137		0.9254		0.9159			0.8893

Source: *Quadros de Pessoal*, $2005-19$.
Note: The table displays the labor income variance decomposition for the three subperiods in consideration and the whole sample (left two columns). The variance decomposition follows equation 4 in the text on the restricted Abowd et al. (1999) specification (equation 1). The decomposition is performed in the largest connected set and in the three-way leave one out sample, which is the extension of the Kline et al. (2020) method for the three high-dimensional fixed effects case.

subperiods considered and the sample as a whole. The worker effects are the most important source of heterogeneity, followed by the firm effects and sorting between workers and job titles. Even if the job title heterogeneity is not as large as the worker or firm heterogeneity, its size is comparable to that of worker-firm sorting. These results are also consistent for the sample as a whole. Interestingly, for the whole sample, worker variance decreases, and worker-firm sorting increases.

Table 3 presents the changes in composition throughout the samples. The last two columns show the changes in the second and third sub-samples with respect to the first period. The last column shows that the negative trend in inequality is totally explained by a decrease in the variance of the firm effect component, the job title, and the covariance of firm and job title effects. These findings add to the literature highlighting the role of firms as major actors in wage inequality dynamics. For Brazil, Alvarez et al. (2018) find that the firm component explains around 39 percent of the decline in wage inequality between 1996 and 2012. In Germany and the United States, the change was led by better workers sorting into better firms (Card et al., 2013; Song et al., 2019). In the next section, we pin down and quantify the channels underlying the compression the firm effect, the job title fixed effect, and the covariance of the firm and job title effects.

Table 3: Changes in the Composition of Wage Variance —Largest Connected Set

	2005-09	2010-14	2015-19		
	(%)	(%)	(%)	Δ_{1-2}	Δ_{1-3}
Variance workers effects	44.01	51.55	48.11	7.542	4.099
Variance firms effects	24.99	21.82	18.78	-3.170	-6.211
Variance job title effects	5.81	5.30	4.91	-0.508	-0.900
Covariance of worker-firm ($2\times$)	4.98	2.83	7.44	-2.154	2.454
Covariance of worker-job title ($2\times$)	10.24	8.38	10.85	-1.861	0.610
Covariance of firm job title ($2\times$)	3.81	4.84	3.73	1.030	-0.084

Sources: *Quadros de Pessoal*, $2005-19$.
Note: The table displays the share of variance explained by each component in the largest connected set sample. The last two columns present the changes between the second and third sub-periods with respect to the first one.

The estimates reported in this section are robust to different sample definitions. There is bias in the variance from the AKM when it is estimated in the largest connected set instead of the *leave-out connected set* sample. The importance of each component in the variance decomposition (equation 4) is sensitive to the sample and the network structure considered. Using Swedish data, Bonhomme et al. (2019)[15] show that the variance of the firm component decreases by almost 40 percent when the variance decomposition is computed in the leave-out sample. Kline et al. (2020) report a 30 percent decrease in the firm component variance when it is estimated in the Veneto matched employer-employee data. These papers show the importance of considering the leave-out sample to achieve a robust calculation of the variance and its dynamics.[16]

In our case, however, implementing the AKM variance decomposition in the *leave-out connected set* is not straightforward. The commonly used *leave-out connected set* assumes only a two-way fixed effect model of wages. If we consider the job title component, the methodology must be extended to a three-way fixed effects model of wages. Although the

[15]See Bonhomme et al. (2019), Table S2 in the Appendix.
[16]Andrews et al. (2008) propose another method for correcting the biases in the AKM framework, which is also known as the trace correction. Yet, the method assumes homoscedasticity. Since we do not assume homoscedasticity, we do not calculate the corrected variance using Andrews et al. (2008)'s correction.

calculation of the largest connected set with multiple high-dimensional fixed effects is well known (Guimaraes and Portugal, 2010), the computation of the three-way *leave-one-out sample* had not yet been implemented in previous literature. Our definition of the *leave-one-out sample* extends the KSS methodology to include worker, firm, and job title fixed effects. The three-way *leave-one-out sample* is constructed under a simple assumption on the network structure: workers are connected to firms, and firms are connected to job titles.[17] As can be seen in Table 2, the differences between the variances in the two samples are not as large as in previous studies. Reassuringly, inclusion of the job title fixed effect allows us to maintain the largest connected sample so that each component of the variance in the *leave-one-out sample* is close to the corresponding component in the largest connected set.

5 Firm Pay Premiums and Firm Characteristics

In this section, after having *robustly* estimated the restricted AKM (Abowd et al., 1999), we project the estimated fixed-effects onto the covariates, and later extract the passthrough to the firm fixed effects variance. Specifically, we project two outcomes. First, we project the fixed effects. This amounts to explaining how the firm covariates change the level (expected first moment) of the firm pay premium distribution. Second, we project the nonparametric variance counterpart, namely, the RIF (Firpo et al., 2009, 2018). This amounts to explaining how the firm covariates change the *dispersion* of the firm fixed effects (the second moment of the distribution). Finally, we decompose changes in the firm pay premium dispersion dynamics into a *return effect* and a *composition effect*. That is, we tease apart changes in the firm pay premium dispersion coming from changes in the composition of firm characteristics and changes in the returns of those characteristics.

To explain the dynamic of the firm pay premiums and their dispersion, we focus on four factors: firm size, firm performance, market power (in the labor and product markets), and firm-exposure to the minimum wage. These factors were chosen based on past literature and are in line with the stylized facts reported in section 4.

Firm Size Consistent with our stylized fact reporting the decline in the large firm wage premium, we posit firm size as a key feature in explaining the firm pay premium. The role of firm size in firm fixed effects has been analyzed in the case of Brazil (Alvarez et al., 2018). Previous literature highlights that working conditions improve with firm size, both in monetary and non-monetary compensation (Bloom et al., 2018). To explain how firm size is linked to pay premiums, several explanations have been advanced. First, size might affect the firm's relative bargaining power and hence affect rent splitting and wages. However, it is also the case that larger firms tend to have larger rents. This means that workers working in larger firms have a bigger total rent to share among themselves. Another explanation is that larger firms pay efficiency wages to maintain and attract more productive workers (Katz, 1986; Krueger and Summers, 1988). This being the case, the average wage is higher in larger firms. Finally, another explanation is that the environment in a

[17] We rule out the less restrictive possibility that workers connect directly to both firms and job titles, since that would decrease the probability of finding articulation points in the network.

larger firm is less agreeable, so firms must compensate workers for this (*compensating differentials*) (Kostiuk, 1990; Bonhomme and Jolivet, 2009; Bloom et al., 2011; Lamadon et al., 2022). In this paper, we use employment within the firm as our measure of firm size.

Firm Performance Firm performance is a natural candidate to explain firm-specific pay premium variability. Changes in productivity directly affect the match surplus value for the firm, which impacts the firm-specific component and wages (see equation 3). However, capturing differences in firm performance at the firm level is known to be challenging. To capture firm performance, we consider two proxies: *value added per worker* and *firm skill requirement*. The former is important since firm productivity affects the extent to which firms share or not rents with their workers (Card et al., 2016).[18] The latter is important since there is a close link between innovation, performance, and the nature of tasks performed inside the firm. For instance, there is abundant evidence showing that technology adoption changes firms' occupational and skill composition (Autor et al., 2003; Autor and Dorn, 2013; Autor, 2015). To capture the organizational dimension of the firm, we consider the firm skill requirement (Acemoglu and Restrepo, 2020) as a proxy for the firm's technology adoption. We propose a skill index that encompasses the average skill requirements of the workforce, which is a metric for knowledge composition in the firm.

To build this skill metric, we closely follow Lise and Postel-Vinay (2020). We start by creating a clean crosswalk between the 2008 International Standard Classification of Occupations (ISCO) and the Standard Occupational Classification (SOC). We then clean O*NET data, to have a crosswalk between each of the 35 skill dimensions (for example, active learning levels, intensity of complex thinking required, and mathematics needed) and SOC codes. Next, we reduce the dimension of this matrix and make it a single vector. That is, we compute the first principal component using principal component analysis (PCA). Equipped with this object, we normalize the principal component such that it is bounded between zero and one.[19] Still using O*NET data, we convert 8-digit SOC codes into 6-digit SOC codes and adjust our skill measure to be the average of each 8-digit measure within each 6-digit code[20]. Then, we merge this information on skills at the SOC occupation level with corresponding ISCO 2008 codes. We trim the ISCO 2008 classification at the 3-digit level and take the mean of the skill measure within each of these 3-digits ISCO 2008 categories. We can bring together the ISCO 2008 data and the Portuguese Classification of Professions. Merging this information with QP yields a measure of skill intensity for each worker in our data. Averaging this measure within the firm, we get a measure of firm skill composition. We expect that firms that are more technologically intensive employ workers with higher skills. Both labor productivity and

[18]We use Card et al. (2016) definition of value added per worker, where we assign 0 to values of value added per worker below the positive rent sharing threshold.

[19]Formally, we call the principal component of each observation p_i. We denote S the set including each non normalized principal component. We normalize each principal component according to

$$n_i = max\left\{\frac{p_i - min\{S\}}{max\{S\} - min\{S\}}; 0\right\}$$

[20]For example, if profession 11111112 has a skill measure of 0.70 and profession 11111120 has a skill measure of 0.76, then profession 111111 will have a skill measure of 0.73. This leaves us with 747 different occupations.

firm skill requirements might affect firms' compensation policy and have effects on labor income inequality (Acemoglu and Restrepo, 2019, 2020, 2021).

Market Concentration The role of imperfect market competition in labor market outcomes has been of great interest lately, and there is evidence that labor market power is depressing wages (Naidu et al., 2018; Naidu and Posner, 2021; Azar et al., 2019). When the relative size of the firm is large with respect to the market in which it operates, the firm might have greater power in wage setting negotiations, leading to lower wages. This can happen if the lack of firm competition undermines the credibility of an outside option for workers[21] or if firms coordinate to set workers' earnings. To capture these dimensions, we use two measures of imperfect market competition. First, we use a measure of market industry concentration using the average Herfindahl Hirschman Index (HHI) computed using firm sales at the 4-digit economic sector level. Second, we use a measure of labor market concentration based on an HHI built using the employment share for each local labor market.[22]

Institutional Factors Section 4 described how different parts of the earnings distribution grew at different rates over the past two decades. This suggests that different demographic and occupational groups might have been affected diversely. We consider institutional factors that are consistent with the decrease in inequality being driven by the lower tail of the earnings distribution. Specifically, we consider the institutional framework for wage bargaining in Portugal, which is composed of a national minimum wage and a set of collective bargaining agreements. The importance of the minimum wage has been widely discussed in economics. Leung (2021) notes how changes in the minimum wage might affect real wage inequality in the United States. This channel has also been considered for the case of Portugal (Portugal and Cardoso, 2006) and more recently for the case of Brazil (Engbom and Moser, 2022). The role of collective agreements in wages and employment has also been the object of discussion. There is evidence of its importance across countries (see Devicienti et al. (2019) and Fanfani (2020) for the case of Italy, Card and Cardoso (2021) for the case of Portugal, and Lagos (2019) for the case of Brazil). In our workhorse specification, equation 1, the job title fixed effect captures heterogeneity in collective agreements. Additionally, we consider the share of minimum wage workers in the firm and evaluate how this characteristic impacts pay premium dispersion and its dynamics.

In the following sections, we investigate how firm size, firm performance, market concentration, and the minimum wage affect the level and dispersion of the identified AKM components − firm, job title, and their covariance. This endeavor will allow us to quantify the overall and individual contributions of these characteristics to wage inequality dynamics. Table A3, in Appendix A, presents summary statistics for these covariates.

[21]Consider the case of two firms operating in the same market. Both firms hire the same type of workers, and if the firm is large enough compared to its competitor, the claim of an outside option from a worker is not credible. In this case, the worker has less wage-negotiation power.

[22]We define a labor market as the 4-digit occupation level at the 2-digit region level. This local labor market definition allow us to compare a specific occupation (e.g., secretaries) in a given region (e.g., Algarve).

5.1 First Moment of the Fixed Effect Distribution

Following Alvarez et al. (2018)'s approach we project the firm fixed effects on our chosen covariates. We repeat this exercise for job title fixed effects, and for the covariance of the firm and job title fixed effects. By doing so, we can pin down the factors that influence the fixed effects on average. For each subperiod P, using firm observations and weighting by the number of worker-year observations, the following model is run by ordinary least squares:

$$\hat{y}_j^P = \alpha^P + \bar{\mathbf{X}}_j^P \Gamma^P + s_j^P + r_j^P + \mu_j^P \tag{5}$$

where $\hat{y}_j$ stands for the estimated firm or job title fixed-effects in each of the subperiods. $\bar{\mathbf{X}}_j$ is a matrix of the average firm characteristics for firm j in each subperiod, and μ_f stands for an independent and identically distributed (i.i.d.) error term. The matrix $\bar{\mathbf{X}}_j$ includes the average size, value-added per worker, industry concentration, labor market concentration, firm skill requirement, and share of workers at the minimum wage. α is the subperiod-specific regression intercept. To proxy value added per worker, we use the average log of gross value-added (at market prices) during the subperiod, while to proxy industry concentration, we use the average HHI computed at the 4-digit economic sector level, for each subperiod. Our specification also accounts for firm sector of activity (at the 2-digit level) fixed effects and firm region (at NUTS II) fixed effects.

In all the regressions, the estimated fixed effects have been re-scaled with respect to the largest firm during the first subperiod, such that this firm fixed effect is zero. We perform this re-scaling to compare firm fixed effect levels over time. The method assigns the value of zero to the base category, so the values can be re-scaled directly from the implementation. Another key concern addressed in all the following regressions is that because the fixed effects are estimated values, they might include sampling error, which could overestimate the variance explained by each individual component and affect inference. In section 7.2, we empirically verify that this error is relatively constant over time (and cross sectionally). On top of this, standard errors are calculated by Efron bootstrap, which leads to a conservative inference (Hahn and Liao, 2021).

5.1.1 Firm Pay Premium

Understanding the factors that determine why different firms pay different wages is essential for understanding wage dynamics. Table 4 reports the resulting coefficients from projecting the estimated firm pay premium into the logarithm of average firm size, average value added per worker, average product market concentration, average labor market concentration measures, and average knowledge (skill) composition. Across subperiods, the predictive power of our model seems to be relatively constant at around 30 percent. The results in the table control for sorting into jobs and job titles, and several aspects deserve special attention: workers who work in firms that are larger, more productive, operate in less competitive environments, and in which the average job skill requirements are higher receive significantly higher wages. The occupational structure of the firm has the largest impact. However, workers who work in monopsonic firms and firms with a high share of minimum wage workers are expected to be paid less.

A striking finding in Table 4 is that the coefficients decrease (in absolute value) from the first to the third subperiod. Alvarez et al. (2018) using this finding for value added

Table 4: Projection of Covariates into Firm Fixed Effects (All Periods)

	$\hat{\psi}_j$ - Firm fixed effects			
	2005-09	2010-14	2015-19	2005-19
Firms size (log)	0.028***	0.020***	0.018***	0.027***
Value added per worker (log)	0.012***	0.011***	0.009***	0.015***
Product concentration	0.089***	0.081***	0.039***	0.087***
Labor market concentration	-0.010***	-0.028***	-0.002***	-0.038***
Share of minimum wage workers	-0.289***	-0.265***	-0.280***	-0.265***
Firm skill requirement	0.401***	0.281***	0.189***	0.375***
N	6,779,289	6,301,025	6,595,153	22,192,961
R^2	0.340	0.311	0.348	0.428

Significance: *p<0.1; **p<0.05; ***p<0.01

Source: *Quadros de Pessoal*, $2005-19$.
Note: The table displays the coefficients obtained when projecting the estimated firm pay premium component into the covariates. All the estimations presented in the table control for sector and region. The standard errors are calculated by bootstrap, using 500 repetitions.

per worker, suggest that the relationship between returns to characteristics and the firm effects flattens over time, which is consistent with the decline of the passthrough from characteristics to pay (wages). We formally test the decline in passthrough below, but it is worth noting this result upfront.[23]

5.1.2 Job Title Pay Premium

Different types of jobs have different compensation schemes, depending on the occupation, sector, skills and tasks performed and the institutional settings that might regulate the profession. Such dynamics are captured by job titles in specification 1. We regress the estimated job title fixed effects on the selected firm characteristics, to understand why different jobs pay differently. The results are presented in Table 5. Some aspects stand out from the results. First, the explanatory power of the firm variables in the firm job title fixed effects is lower compared with the size of the resulting coefficients when projecting the firm pay premium (see Table 4). Second, it also seems that in this case the coefficients shrink over time, which suggests that there is a decline in the passthrough. As for the previous case, we quantify this change in the next subsection. Nevertheless the signs of the coefficients coincide for all the variables except the market concentration variables: the job title pay premium increases in jobs that are in concentrated labor markets, while it decreases in markets that have more

[23] To compare our findings with previous literature, we evaluate the decline in the coefficient of value added per worker graphically. The results are shown in panel (a) in Figure A9, in Appendix A. We have verified that this decline in the passthrough is not the result of changes in the firm size distribution over time. When we evaluate this relationship conditional on firm size (measured by the average number of workers in each subperiod), we verify that the decline in value added passthrough is common for all the firm size groups.

competition. This could be driven by the workers' ability to organize themselves and use the institutional tools that the collective agreement provides to increase their negotiation power.

Table 5: Projection of the Covariates into Job Title Fixed Effects (All Periods)

	$\hat{\phi}_j$ - Job title fixed effects			
	2005-09	2010-14	2015-19	2005-19
Firms size (log)	0.004***	0.002***	0.000***	0.002***
Value added per worker (log)	0.001***	0.001***	0.002***	0.003***
Product concentration	-0.037***	0.021***	0.011***	-0.001
Labor market concentration	0.057***	0.030***	0.007***	0.040***
Share of minimum wage workers	-0.023***	-0.013***	-0.018***	-0.031***
Firm skill requirement	0.355***	0.229***	0.251***	0.455***
N	6,779,289	6,301,025	6,595,153	22,192,961
R^2	0.276	0.234	0.224	0.297

Significance: *p<0.1; **p<0.05; ***p<0.01

Source: *Quadros de Pessoal*, $2005-19$.
Note: The table displays the coefficients obtained when projecting the covariates into the estimated job title pay premium component. All the estimations presented in the table control for sector and region. The standard errors are calculated by bootstrap, using 500 repetitions.

Moreover, Table 5 provides evidence that if the workforce composition in the firm is highly skilled, it increases the job title pay premium. It is then relevant that the workforce structure is associated with the task intensity and degree of technology used by the firm. It is interesting to note how this factor passes to wages through the types of jobs held by the workers. Moreover, as almost all the variable coefficients are positive, we might infer that there is positive sorting between good firms and good job titles. The variance decomposition in Table 2 shows that this is indeed the case.

5.2 Second Moment of the Fixed Effect Distribution

Section 5.1 focused on the mean of the distribution of fixed effects. In this section, we are interested in the degree of dispersion of the distribution, identified by the variance —the second moment of the distribution. To see the changes in the variance and isolate the role of firm fundamentals in the passthrough or change in structure, it is common in the literature to apply a two-step procedure.[24] The result computes the variance of the fixed effects explained by observable characteristics. Instead of relying

[24]To calculate the variance, the estimated coefficients $\mathbf{b}$ are multiplied by the variance of the design matrix $\mathbf{X}$: $\text{var}(\hat{y}) = \boldsymbol{b}'\text{var}(\mathbf{X})\boldsymbol{b}$. To compare the change in the passthrough over time the design matrix is held constant, and to see the change in observable characteristics, $\mathbf{b}$ is held constant. The counterfactual exercise allows us to determine the source of the decline.

on a post-estimation procedure, we propose to project the covariates of interest into the nonparametric variance counterpart, which is also known as the RIF (Firpo et al., 2009, 2018). This methodology allows us to estimate directly how the observables affect the dispersion of the variable of interest, which in this case is how firm characteristics affect the firm pay premium variance, job title pay premium variance, and their covariance. More importantly, we can quantify the impact of each firm observable and divide the effect into the passthrough and structural effects. In this way, we can identify the drivers of the wage inequality dynamics. To calculate the nonparametric version of the variance, we use the following:

$$\mathrm{RIF}_{(\sigma_x^2),j} = \left(x_j - \int x dF(x)\right)^2$$

For the covariance we use the following,

$$\mathrm{RIF}_{(\sigma_{x,y}^2),j} = \left(x_j - \int x dF(x)\right)\left(y_j - \int y dF(y)\right)$$

Intuitively $\mathrm{RIF}_{(\sigma_x^2),j}$ and $\mathrm{RIF}_{(\sigma_{x,y}^2),j}$ calculate the marginal contribution of observation j to each statistic. Thus, and following Firpo et al. (2009), for each subperiod P, we can estimate the change in the dispersion by a change in the firm characteristics.

$$\sigma_{\hat{y}}^{2P} = \mathrm{RIF}^P_{(\sigma_{\hat{y}}^2),j} = \bar{\mathbf{X}}_j^P \beta_j^P + s_j^P + r_j^P + v_j^P \tag{6}$$

where $\mathrm{RIF}_{(\sigma_{\hat{y}}^2),j}$ stands for the individual contribution to the variance of the estimated firm or job title fixed effects in each subperiod. $\bar{\mathbf{X}}_j$ is a matrix of the average firm characteristics for firm j in each subperiod, and v_j stands for an i.i.d. error term. As before, matrix $\bar{\mathbf{X}}_j$ includes the firm characteristics that are relevant for our exercise, and our specification also accounts for firm sector and region fixed effects. Subsection 5.2.1 reports the findings on the dispersion; the tables reporting the firm fixed effects (Table A4), job title fixed effects (Table A5), and their covariance (Table A6) are in Appendix. Subsection 5.2.2 decomposes the variance over time and divides the contributions of individual, single observables into pass-through and structural effects.

5.2.1 Fixed Effects Dispersion

Table A4 presents the results of regressing firm characteristics on the contribution of firm pay premium variance. The regressions on the dispersion of the firm pay premium suggest that as firm size increases, the pay premium decreases, that is, there is less heterogeneity in the firm pay premium between larger firms than between smaller firms. Instead, there is increased heterogeneity in all the other variables. In firms that have larger productivity per worker, compete in less competitive markets, have more local labor market concentration, have a larger share of workers earning the minimum wage, and have a more specialized workforce, there is an increase in the dispersion of the firm pay premium. Similar to the results for the first moment of the distribution, the coefficients decrease (in absolute value) from the first to the third subperiod. This result implies that in the hypothetical case in which the structure of the covariates does not change across periods, the passthrough of the firm characteristics to the firm pay premium falls from the first to the third sub-period.

The correlations of the covariates with the job title pay premium variance are all small and positive. The exception is product market concentration, which is not significant. Table A5 provides support for how the firm characteristics increase dispersion. Although the resulting coefficients are small, the proportion of the variance explained by the model is between 8 and 19 percent. Unlike the firm pay premium, not all the coefficients decrease between the first and third periods, so the preliminary analysis of the passthrough is inconclusive. In this case, product concentration flips sign and increases, and size does not change.

An advantage of using the non parametric version of the covariance is that we can calculate the effects of firm characteristics directly in the covariance. So we can explain which factors increase or decrease the assortativity between good firms and good jobs. Table A6 presents the results of the projection of the selected covariates into the covariance. The proportion of the variance explained by the firm characteristics in this case is larger than that in the two previous regressions, and the model explains between 12 and 19 percent of the total variation. The magnitudes of the coefficients, as for the job title pay premium are smaller than those for the firm pay premium. In this case, we cannot perform a preliminary exercise to determine the importance and direction of the passthrough.

One of the purposes of this section is to quantify the importance of and identify the firm characteristics that drove the decrease in the firm pay premium, job title pay premium, and their covariance. Even if the results from the second moment elucidate the variables that are more important in increasing pay for different pay premium dispersions, they are not sufficient to assess what was the principal driver of the reduction in inequality.

5.2.2 Passthrough and Composition

To decompose the change in variance throughout the period in consideration, we use an Oaxaca-Blinder decomposition (Oaxaca, 1973; Blinder, 1973; Kitagawa, 1955; Card et al., 2016; Firpo et al., 2018). The decomposition allows us to identify how much of the reduction over time in the fixed effects was due to changes in the distribution of the covariates, and the amount of the decrease that was due to a reduction of the passthrough from the firm characteristics to the pay premiums. The following equation is used for the decomposition:

$$\hat{\Delta}^{\sigma^2} = \underbrace{(\bar{\mathbf{X}}_1 - \bar{\mathbf{X}}_0)'\hat{\beta}_0}_{\text{Composition Effects}} + \underbrace{\bar{\mathbf{X}}_1'(\hat{\beta}_1 - \hat{\beta}_0)}_{\text{Pass-through}} \tag{7}$$

where $\hat{\Delta}^{\sigma^2}$ is the total change in the variance, where the composition effects are calculated by fixing the returns from the characteristics and changing the distribution of the design matrix over time. The passthrough is calculated by fixing the design matrix and calculating the changes in the returns from the characteristics.

The results of the decomposition are presented in Table 6. To calculate the standard errors, we bootstrap the whole procedure using 500 replications. We follow Hahn and Liao (2021) to perform the bootstrapping. The top section of the table shows the aggregate decomposition, for each of the fixed effects that lead to the decrease in inequality. The first two columns assess the reduction in the variance of the firm pay premium. Columns 3 and 4 decompose the change in the job title pay premium and the last two columns decompose the decrease in sorting between firms and job titles. The bottom section of the table provides the details of the decomposition and how each firm covariate contributes to the reduction in inequality in each component. For all the fixed effects, the driver of the decrease in the pay premium is due to a decrease in the passthrough. In all cases, the passthrough overcompensates the increase in variance of the distribution of the characteristics.

Focusing on the passthrough, from the last to the first subperiod, all the covariates except the logarithm of size, the intercept, the region, and the sector, explain the compression in the dispersion of the firm pay premium. In particular, both firm performance proxies are drivers of the fall in inequality dynamics. The first decomposition considers the change that occurred before the financial crisis, which could be the reason why the signs of some of the variables change, specifically for value added and firm size.

In the case of the dispersion in the job title pay premium, all the covariates except the intercept, product market concentration, and region contribute to the decrease in the passthrough. The four most important components that drive the fall in the passthrough are value added, labor market concentration, occupational structure of the firm, and sector. In this component, the sector is the main driver of the fall of the passthrough. When we consider the change in the distribution of firm observable characteristics, it contributes to an increase in the dispersion. However, this contribution is rather small and the passthrough dominates the overall effect.

Considering the assortativity between the firm and job title components, its decrease was due to the role of firm performance (both value added and occupational structure) along with the contribution of sectors. Looking at structural changes over subperiods, the contribution of the distribution of characteristics is small and increases the dispersion. In this case, the variable that drives the fall in assortativity is the firm sector.

6 Mediation Analysis

Consistent with Alvarez et al. (2018), our analysis in the previous sections hinges on the assumption that changes in firm characteristics lead to changes in the level and dispersion of firm pay, fully mediated by their effect on firm pay premiums. This ripple effect from characteristics into wages is indirect, to the extent that the firm pay premium is the sole mediator between firm characteristics and observed wages.

6.1 Strategy

In this section, we relax this assumption and consider a more flexible modeling framework instead. We inquire how firm characteristics affect wages, both through the firm pay premium (*indirect effect*) and independently of this mediator (*direct effect*). We represent these competing frameworks by means of simple diagrams in Figure 2. In the left panel, we see that the firm effect captures all the changes in the variable of interest, and then passes them to the wage directly. This is the framework we have implicitly assumed in section 5.1.1. The right panel shows instead the case in which

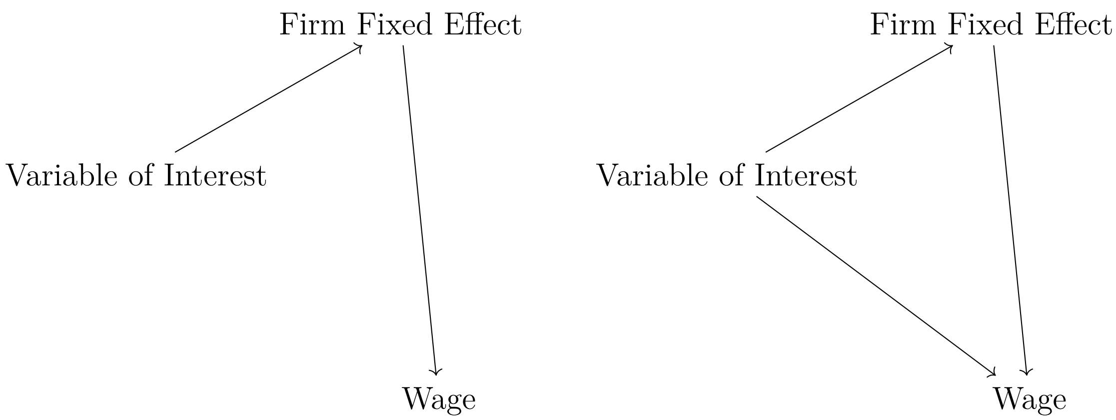
Table 6: Oaxaca-Blinder Decomposition for Changes in the Firm Fixed Effects Variance (RIF)

	Firm fixed effects		Job title fixed effects		Cov(JT,F) fixed effects	
	$\Delta\sigma^2_{\psi(1.2)} \times 100$	$\Delta\sigma^2_{\psi(1.3)} \times 100$	$\Delta\sigma^2_{\phi(1.2)} \times 100$	$\Delta\sigma^2_{\phi(1.3)} \times 100$	$\Delta\sigma^2_{(\psi,\phi)(1.2)} \times 100$	$\Delta\sigma^2_{(\psi,\phi)(1.3)} \times 100$
Total effects	-0.869	-2.379	-0.244	-0.529	-0.109	-0.296
Passthrough	-1.097	-2.807	-0.444	-0.799	-0.193	-0.435
Composition effects	0.228	0.427	0.201	0.270	0.084	0.139
Total effects						
Firms size (log)	-0.342***	0.475***	0.098***	-0.091***	-0.015***	-0.023***
Value added per worker (log)	0.553***	-1.203***	-0.867***	-0.577***	-0.508***	-0.394***
Product concentration	-0.163***	-0.165***	0.238***	0.378***	0.095***	0.121***
Labor market concentration	-0.096***	-0.087***	-0.318***	-0.562***	-0.111***	-0.214***
Share of minimum wage workers	-0.423***	-0.443***	-0.043***	-0.064***	-0.107***	-0.122***
Firm skill requirement	-2.235***	-3.471***	-0.988***	-0.198***	-0.414***	-0.306***
Region	0.006	-0.031**	-0.07***	0.062***	-0.008***	0.029***
Sector	1.758***	0.607	-0.94***	-1.942***	-1.213***	-1.255***
Passthrough						
Intercept	0.073	1.938***	2.646***	2.466***	2.172***	1.869***
Firms size (log)	-0.241***	0.637***	0.066***	-0.141***	-0.028***	-0.044***
Value added per worker (log)	0.573***	-1.214***	-0.855***	-0.583***	-0.501***	-0.398***
Product concentration	-0.17***	-0.177***	0.239***	0.378***	0.096***	0.122***
Labor market concentration	-0.101***	-0.096***	-0.35***	-0.614***	-0.123***	-0.234***
Share of minimum wage workers	-0.582***	-0.858***	-0.061***	-0.111***	-0.141***	-0.213***
Firm skill requirement	-2.562***	-3.793***	-1.107***	-0.316***	-0.472***	-0.363***
Region	0.017	0.003	-0.065***	0.076***	-0.006**	0.035***
Sector	1.896***	0.754*	-0.958***	-1.954***	-1.189***	-1.21***
Composition effects						
Firms size (log)	-0.101***	-0.163***	0.031***	0.05***	0.013***	0.022***
Value added per worker (log)	-0.021***	0.011***	-0.012***	0.006***	-0.007***	0.004***
Product concentration	0.007***	0.012***	0.000*	0.000*	-0.001***	-0.002***
Labor market concentration	0.006***	0.009***	0.032***	0.052***	0.012***	0.019***
Share of minimum wage workers	0.159***	0.416***	0.018***	0.047***	0.035***	0.091***
Firm skill requirement	0.327***	0.322***	0.119***	0.118***	0.057***	0.057***
Region	-0.011***	-0.033***	-0.006***	-0.014***	-0.002***	-0.006***
Sector	-0.138***	-0.147***	0.018***	0.012***	-0.024***	-0.045***

Significance: *p<0.1; **p<0.05; ***p<0.01

our variables of interest directly affect the outcome variable, but part of the effect also passes through the firm fixed effect. In this latter setting, we ask: *does the firm pay premium still play a significant role in explaining the swift decline in wage inequality?*

Figure 2: Mediation Alternatives for the Firm Pay Premium



(a) Indirect effect

(b) Indirect + Direct effect

Note: In panel (a), we depict a setting where the firm effect captures all the changes induced by the variables of interest (for instance, size or concentration), and then passes them on to the wage directly. Panel (b) shows instead the case in which our variables of interest directly affect the outcome variable, but part of the effect also passes through the firm fixed effect.

To tackle this question, we use a form of *mediation analysis*. Mediation analysis allows us to assess how the firm characteristics of interest – such as size, concentration or productivity – affect the level and dispersion of wages both directly and through a set of mediating variables, such as firm or worker fixed effects. To this end, we leverage the decomposition proposed in Gelbach (2016). The decomposition starts with a restricted baseline specification to which covariates are subsequently added. Thereafter, by using the omitted variable bias (OVB) formula, researchers can pin down the contribution of adding or removing each covariate for the change in a coefficient of interest, where the addition of covariates is sequence independent. The Gelbach decomposition is suitable for our case, since it can easily accommodate high dimensional fixed effects.[25] We use this approach to show how firm characteristics can affect wages and wage dispersion both directly and indirectly. They affect wages directly through OVB, but indirectly via the mediator variables (in our case, the fixed effects).

To see this more clearly, consider a general restricted specification (”base” model) that contains the variable(s) of interest (such as size or labor marker concentration), and a set of controls (in the limit, a constant alone):

$$Y = X'\gamma + G\beta_0 + \epsilon \tag{8}$$

[25] This approach has been used recently in various fields of economics. In terms of methodology, our approach is similar to that of Cardoso et al. (2016) in their study of the gender wage gap. The authors assess the contribution of a set of high-dimensional fixed effects to the omitted variable bias of the gender wage gap. Other recent papers use the Gelbach decomposition to study unionization (Addison et al., 2022), wage persistence (Carneiro et al., 2022), and displaced workers losses (Raposo et al., 2021).

where X is the design matrix of the controls, and γ are returns to those characteristics. The matrix G is the matrix of the variables of interest, for which we want to calculate the contribution of some omitted variables. β are returns to those characteristics and ϵ is an error term. Now consider a full specification ("full" model) that incorporates additional variables. In our case, the additional variables, are contained in $H\xi$[26].

$$Y = X'\gamma + G\beta_1 + H\xi + \epsilon \tag{9}$$

Define a matrix A_G as $A_G = (G'M_X G)^{-1}G'M_X$, where M_X is the annihilator matrix[27]. After some matrix manipulation, it can be shown that

$$\beta_0 - \beta_1 = A_G(H\hat{\xi}) = \tau_\xi \tag{10}$$

where (τ_ξ) are the mediation components. In our case, we will have one component for each fixed effect in our model. Each of these components captures how each firm characteristic of interest (such as size or concentration) passes through each of the fixed effects components into the output.

In this section, we consider the AKM model described by equation 11 as the *full model*, in which the outcome variable, y_{it}, the wage or its variance, is linear additive on time-variant characteristics, worker, firm, time, and job title fixed effects.

$$y_{it} = \underbrace{\beta x_{ijt} + \tau_t}_{\approx \textit{ base model}} + \underbrace{\alpha_i + \psi_{j(i,t)} + \phi_{k(i,t)}}_{\text{mediation components}} + \epsilon_{it} \tag{11}$$

We use the Gelbach methodology to decompose the mediating effects for worker, firm, and job title fixed effects in model 11. Our concern is that when including the direct effect, there is the possibility that the pass through could become irrelevant. We test this as follows. First, we estimate a version of equation 11 that does not include the mediation components and we recover the estimated coefficients for our variables of interest (value added per worker, for example). Second, we estimate equation 11 with the mediation components (*i.e.* with the worker, firm and job title fixed effects) and we recover the estimated coefficients for our variables of interest. Equipped with these two sets of coefficients, we then evaluate how much do coefficients change after we include the firm fixed effects. We break down this change in our coefficients of interest into the contribution of job, firm and worker fixed effects.

In an additional exercise, we repeat this procedure for our different subperiods, and evaluate how the contribution of worker, firm and job title fixed effects changes over time. This amounts to quantifying how the indirect and direct channels evolve over time. This is relevant for the following reason. While Table 6 illustrates the dynamic relationship between the variance of the firm pay premium and the covariates of interest, it does not provide insight into the extent to which these factors affect workers' wages. Instead, in this section, our analysis of the dynamic mediation process over time sheds light on how the characteristics pass through the firm pay premium to wages. We are then able to identify how much of the dispersion of wages has evolved over time due to the firm pay premium indirect effect.

[26] In our setting, the additional variables are the fixed effects for workers, firms, and job titles.

[27] For details on the construction of the matrix, see the appendixes in Gelbach (2016) and Addison et al. (2022).

6.2 Mediation Analysis Results

In Table A7, in Appendix A, we present the results of our strategy using the level of wages as the outcome. In the first column we show the set of coefficients obtained from estimating the restricted model (*i.e.* a wage model omitting all fixed effects). The second column shows the results of estimating the full model (*i.e.* a wage model including all fixed effects). The third column shows how much each coefficient changes going from the restricted to the full model. The fourth, fifth and sixth columns show how much each fixed effect contributed to each coefficient change going from the restricted to the full model. Notice that by construction the sum of the contribution of all fixed effects must add up to the change reported in the third column. This table shows that the inclusion of the direct effect does not render irrelevant the link between firm characteristics and wage level as mediated by the firm fixed effect. For instance, while the coefficient associated with value added per worker decreased by 0.025 (plummeting from 0.027 to 0.002) going from the restricted to full model, 36 percent of this decline was due to the inclusion of firm fixed effects. This indicates that firm fixed effects have an important, though partial, mediation role for value added per worker.

In Table A8, in Appendix A, presents analogous results using wage dispersion as the outcome variable. As before, in the first column we show the set of coefficients obtained from estimating the restricted model. The second column shows the results of estimating the full model. The third column shows how much each coefficient changes going from the restricted to the full model. The fourth, fifth and sixth columns show how much each fixed effect contributed to each coefficient change going from the restricted to the full model. This table indicates that while wage dispersion increases unconditionally with firm skill requirements (1.179) and labor productivity (0.019), the opposite happens once we control for worker, job and firm heterogeneity. In this case, wage dispersion decreases both with firm skill (-0.011) requirements and labor productivity (-0.001). That is, the effect of labor productivity and skill requirement on wage inequality decreased by 0.020 and 1.190 respectively after controlling for all three fixed effects. For labor productivity, the table shows that 26.3 percent of this decline is accounted by firm specific pay policies, while 15.7 percent is due to job title heterogeneity and 58 percent is due to worker heterogeneity.

Overall, this first set of results shows that including the mediating variables absorbs some of the effect of firm covariates that passes trough to the wage and to wage dispersion. However, even when we include the direct effects of the firm characteristics, the mediators play an important role. The signs of the contribution of the variables of interest to the firm pay premium (section 5.1.1) and its dispersion (section 5.2.1) coincide with the mediation sign that we get when performing the mediation analysis.

Table 7 shows the results of implementing the same Gelbach decomposition using the wage level as a dependent variable in equation 11, this time by subperiod. For sake of parsimony, we report only the mediation portion of the firm fixed effect in each subperiod. But, the mediation share of the worker fixed effect and the job title fixed effect by subperiods are available from the authors upon request. The last column in this table reports the decomposition for the entire period and is thus identical to the firm fixed effect contribution reported in column (5) in Table A7, in Appendix A. For

Table 7: Mediating Role of Firm FE to the Wage Level

	$\Delta_{1,3}$	Gelbach - Firm pay premium mediation to wage level			
		2005-09	2010-14	2015-19	2005-19
	(1)	(2)	(3)	(4)	(5)
Firms Size (log)	< 0	0.0065	-0.0030	-0.0105	-0.0114
		(0.0001)	(0.0001)	(0.0000)	(0.0000)
		13.52%	*-10.60%*	*-47.68%*	*-35.41%*
Value Added per Worker (log)	< 0	0.0092	0.0095	0.0080	0.0089
		(0.0001)	(0.0000)	(0.0000)	(0.0000)
		36.01%	*34.11%*	*25.79%*	*32.76%*
Product Concentration	< 0	0.1812	0.2445	0.0892	0.1447
		(0.0009)	(0.0011)	(0.0009)	(0.0004)
		47.44%	*49.44%*	*19.67%*	*32.30%*
Labor Market Concentration	> 0	-0.0048	-0.2299	-0.1841	-0.0852
		(0.0090)	(0.0086)	(0.0074)	(0.0036)
		1.39%	*42.73%*	*32.71%*	*18.22%*
Share of Minimum Wage Workers	> 0	-0.1131	-0.1265	-0.1137	-0.1125
		(0.0004)	(0.0004)	(0.0003)	(0.0002)
		20.42%	*22.85%*	*32.71%*	*20.35%*
Firm Skill Requirement	< 0	0.3855	0.2367	0.1515	0.2099
		(0.0012)	(0.0011)	(0.0009)	(0.0005)
		22.12%	*17.95%*	*13.74%*	*15.44%*

Source: *Quadros de Pessoal*, $2005-19$.
Note: This table presents the pass through of firm characteristics to the level of wages, as mediated by firm fixed effects. See the text for further detail.

each firm characteristic, and for each time interval, we present in italic the percent contribution of the inclusion of firm fixed effects to the change in that characteristic's coefficient going from the restricted to the full model. We indicate in our first column whether this percent contribution is increasing or decreasing over time. Notably, the mediating role of firm pay policies on firm size has reversed over the subperiods: it was positive for the first subperiod, and became negative afterward. The mediating role of firm pay policies on firm size also became more prominent in absolute value (going from 14 percent to -48 percent). The firm pay premium mediating effect has decreased considerably over time for value added, product concentration, and firm skill requirement. Somewhat remarkably, the mediating role of firm pay premiums for both labor market concentration and exposure to the minimum wage has increased substantially. Overall, this table illustrates how our main findings are robust across different partitions of the data, and provides support for the generalizability of our results.

Table 8 shows the analogous results using the wage dispersion as a dependent variable in equation 11. Once again, the coefficients shown here in column (5) match those

Table 8: Mediating Role of Firm FE to Wage Dispersion

	$\Delta_{1,3}$	Gelbach - Firm pay premium mediation to wage dispersion			
		2005-09	2010-14	2015-19	2005-19
	(1)	(2)	(3)	(4)	(5)
Firms size (log)	< 0	0.0072	0.0060	-0.0022	-0.0042
		(0.0001)	(0.0001)	(0.0000)	(0.0000)
		22.94%	*30.97%*	*-14.46%*	*-19.87%*
Value added per worker (log)	< 0	0.0051	0.0061	0.0058	0.0047
		(0.0001)	(0.0001)	(0.0001)	(0.0000)
		27.69%	*30.52%*	*24.59%*	*24.20%*
Product concentration	< 0	0.2959	0.2404	0.1195	0.1925
		(0.0012)	(0.0012)	(0.0010)	(0.0004)
		78.28%	*43.86%*	*22.90%*	*40.43%*
Labor market concentration	> 0	-0.2237	-0.2906	-0.3540	-0.1888
		(0.0118)	(0.0099)	(0.0084)	(0.0036)
		40.14%	*42.33%*	*53.34%*	*29.89%*
Share of minimum wage workers	> 0	0.0947	0.0319	0.0487	0.0393
		(0.0006)	(0.0005)	(0.0004)	(0.0002)
		26.20%	*13.73%*	*34.70%*	*17.60%*
Firm skill requirement	< 0	0.3333	0.2611	0.1806	0.2409
		(0.0015)	(0.0013)	(0.0010)	(0.0005)
		21.93%	*22.22%*	*19.22%*	*20.44%*

Source: *Quadros de Pessoal*, $2005-19$.
Note: This table presents the pass through of firm characteristics to the dispersion of wages, as mediated by firm fixed effects. See the text for further detail.

reported in column (5) in Table A8, in Appendix A, by construction. We indicate in our first column whether this percent contribution is increasing or decreasing over time. As was the case for the level of wages, the mediating role of firm pay policies for firm size on wage dispersion has reversed over the subperiods. It was positive for the first and second subperiods, and became negative in the final subperiod. For labor market concentration, this effect is negative and increasing in absolute value over time: increases in labor market concentration reduce wage variance via firm pay premiums and they do so more and more intensely over time. At the same, the mediating role of firms for product concentration, low-wage workers, and firm skill requirement in wage dispersion has halved. These declines could explain a large decrease in wage dispersion driven by the firm pay premium.

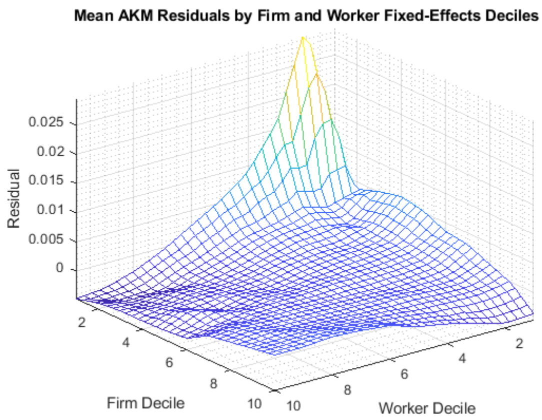
7 Robustness and Threats to Identification

In this section, we address two threats to the validity of our empirical design. First, we provide supporting evidence for the exogeneity mobility assumption imposed in the AKM model. Second, we estimate the omitted variable bias associated with estimated firm fixed effects and show that it is constant cross-sectionally and across time.

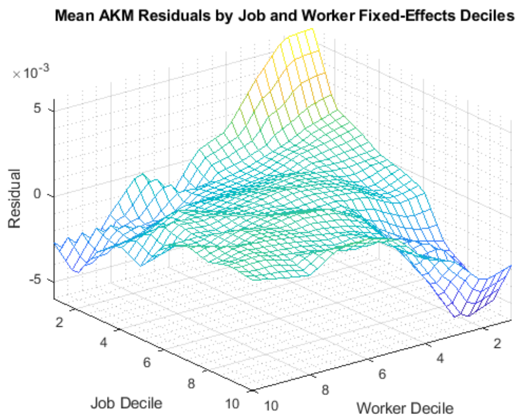
7.1 Exogenous Mobility Assumption

Since the identification of the AKM model derives from workers switching between firms and job titles, Table A1, in Appendix A, presents statistics on the fraction of switchers in each subperiod. The degree of labor mobility is high in Portugal, with around 20 percent of the population switching firms and around 40 percent switching job titles at some point during each subperiod. On average, during the five years in each subperiod, each worker worked in 1.2 firms. This value is around 1.6 for job titles.

Figure 3: AKM Residuals by Firm, Worker, and Job Title Fixed Effect Deciles (First Period).



(a) Firm FE *vs.* worker FE



(b) Job title FE *vs.* worker FE

Source: *Quadros de Pessoal*, 2005 – 19.
Note: Panels (a) and (b) show the mean residuals per firm and worker fixed effect deciles (panel (a)), but also per job and worker fixed effects (Panel (b)) for the first sub-period. All the fixed effect estimates come from the AKM model 1 estimated on the largest connected set. In Figure A5, in Appendix A, we present this exercise for the second sub period, and in Figure A6 we present this exercise for the third sub period.

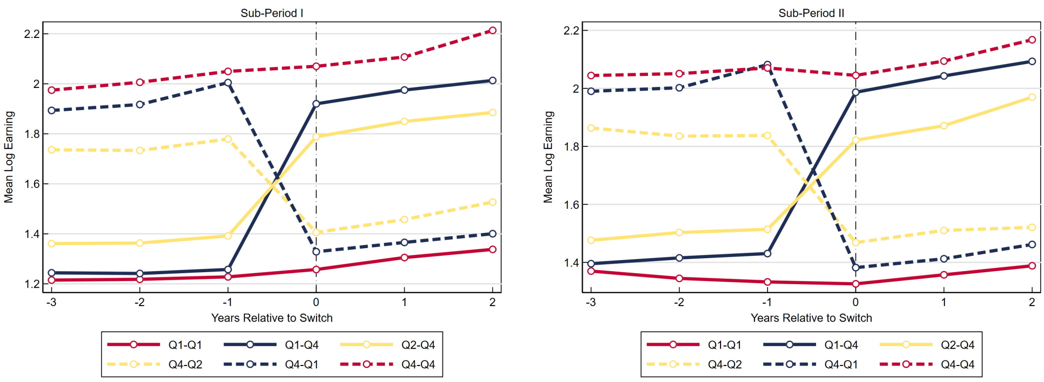
Moreover, the validity of the AKM results presented in section 2 requires that the match effect is unrelated to firm and worker components, that is, the error term is indeed strictly exogenous. With this in mind, and in line with the proposal of Card et al. (2013), we look at the average estimated residuals by firm and worker fixed effect deciles. Figure 3 shows the mean residuals per firm and worker fixed effect decile, but also per job and worker fixed effect for selected subperiods.[28] In support of the AKM hypothesis, the residuals are close to zero, regardless of worker and firm deciles, or worker and job title deciles. Although the residuals appear larger for lower worker and firm fixed effects combinations, the magnitude is still fairly small – less than 0.04. These results support the specification proposed in equation 1.

The second exercise we run to test for exogenous mobility follows Alvarez et al. (2018) and Card et al. (2013). The idea is the following: under AKM assumptions, the gains

[28]The results are similar for all three subperiods.

from those switching, for example, from the first to the forth quartile of firm fixed effects, should be identical to the losses of those making the opposite switch, for example, from the fourth to the first quartile of firm fixed effects. Figure 4 shows the average evolution of earnings of workers changing employers in the first (2005-09) and second (2010-14) subperiods. Consistent with AKM assumptions, indeed workers moving from the first firm quartile to the fourth firm quartile exhibit gains that are similar and symmetric to those moving from the fourth to the first (see the solid and dashed blue lines in Figure 4). These results hold more generally across subperiods and across quartile combinations.

Figure 4: Change in Wages of Workers Moving across Firm Quartiles.



(a) First subperiod

(b) Second subperiod

Source: *Quadros de Pessoal*, $2005-19$.
Note: This figure presents for the first and second sub-periods the average evolution in earnings of workers moving across quartiles of firm estimated AKM fixed-effects. To do these computations, we consider only workers who changed their jobs at most once during each sub-period. The solid blue line, for instance, presents the year-on-year average of log hourly wages of workers having moved from the lowest quartile to the highest quartile, where years are normalized in reference to the event. In this exercise, we require that workers are present at least two years prior and two years posterior to the job switch. Figure A3, in Appendix A, presents these figures for subperiod three, and for the entire time span.

7.2 Estimation Bias

In section 5 we introduced the two-step procedure conducted by Alvarez et al. (2018) and Song et al. (2019). The procedure starts by estimating a restricted wage model, in the vein of Abowd et al. (1999), which can be written in matrix notation as

$$w_t^{i,j,f} = D\delta^{f,P} + F\omega^{i,P} + B\tau^{j,P} + T\phi_t + \epsilon_t^{i,j,f} \tag{12}$$

This equation is in everything equal to the one presented in section 5. This model is restricted in the sense that it omits time-varying characteristics. In the second step, the generated fixed effect values are used to determine the factors that influence on average the fixed effects and to decompose the variance of wages using the variance of these generated coefficients. However, since $\hat{\delta}^{f,P}$ is a generated value, it is estimated with a certain degree of uncertainty:

$$\hat{\delta}^{f,P} = \delta^{f,P} + u_f^P \tag{13}$$

Since $\hat{\delta}^{f,P}$ is used as a dependent variable in the second step of our procedure, our second step parameters of interest will be estimated with an OVB. If the mean and variance of the OVB change across subperiods, this might affect the calculation of the variance. In this case, the bias could affect the validity of our conclusions. To address this concern, we propose directly calculating the OVB. Then, we evaluate whether u_f^P is constant both cross-sectionally and across subperiods.

To compute this bias, we leverage the Frisch-Waugh-Lovell theorem (FWL) and the OVB formula. We start by defining a matrix of form $\Gamma = F\omega^{i,P} + B\tau^{j,P} + T\phi_t$. The annihilator matrix associated to Γ is defined as $M_\Gamma = I - \Gamma(\Gamma'\Gamma)\Gamma'$. Using the FWL theorem, we can rewrite the unrestricted AKM model as:

$$M_\Gamma w_t^{i,j,f} = M_\Gamma D\delta^{f,P} + M_\Gamma X\beta \tag{14}$$

where X is the initially left out vector of time-varying observables. Using the partitioned inverse result and imposing $\hat{\beta} = 0$, we can write $\hat{\delta}^{f,P}$ as

$$\hat{\delta}^{f,P} = (D'M_\Gamma D)^{-1}D'M_\Gamma w_t^{i,j,f} \tag{15}$$

Combining these equations, we can identify the bias u_f^P in:

$$\hat{\delta}^{f,P} = \delta^{f,P} + \underbrace{(D'M_\Gamma D)^{-1}D'M_\Gamma X\beta}_{u_f^P} \tag{16}$$

The conditions under which this bias is null or at least constant cross-sectionally and through time are addressed in meta-analyses. If u_f^P is shown to be constant across both dimensions, then we have $u_f^P = u$. This constant will be absorbed by the constant in the second step, and the bias will therefore be innocuous. To show that this term is constant across sub-periods and constant cross-sectionally, we estimate u_f^P using a random sample of 2,000 firms. The results show that the mean of the bias is constant across sub-periods at around 1.5 and the standard deviation within sub-periods is small and constant at around 0.1. This implies that the mean and variance of the OVB are constant over time. The OVB will therefore have no harmful effect on the computed variance nor our fixed effect estimates.

8 Conclusion

Using a rich combination of linked employer-employee administrative data, we examined the channels through which firms affect earnings inequality dynamics. In the absence of firm-specific effects, the worker and sorting components would have driven up inequality, in line with what happened in other advanced economies. The firm-specific pay policy dispersion, the job title pay premium, and their covariance were responsible for the sharp decline in wage inequality. In our conceptual framework, firm pay concentration depends not just on the distribution of firm characteristics (*a composition effect*), but also on a scaling term that dictates the extent to which the dispersion of those characteristics effectively translates into dispersion of the firm pay premium (*a passthrough effect*). Using an Oaxaca-Blinder decomposition, we found that the reduction in the premiums of firm characteristics, rather than changes in their distribution, drove the decline in earnings inequality.

We quantified the contributions of firm size, performance, labor market concentration, and the share of workers earning the minimum wage for changes in firm pay premium dispersion. We found that value added and the firm skill requirement contributed positively to the dispersion of firm fixed effects. Moreover, we found that these two factors were the main contributors to the fall in the firm pay premium dispersion, and this effect came from a fall in the passthrough from firm characteristics to pay. Our findings suggest that policies that limit product market concentration or foster technology adoption for low-technology firms may be effective in addressing inequality.

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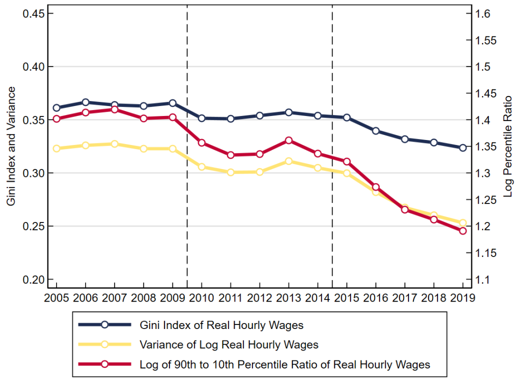
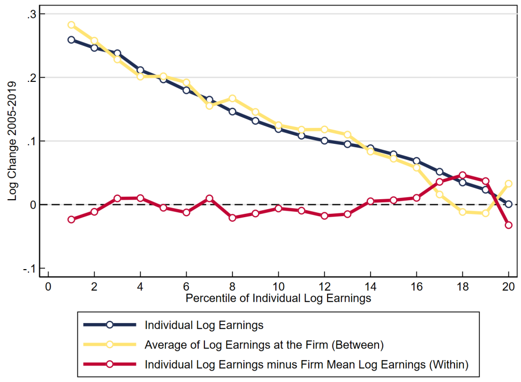
A Additional Figures and Tables

This appendix presents further evidence and clarifications that are not contained in the body of the paper but might be of interest to the reader.

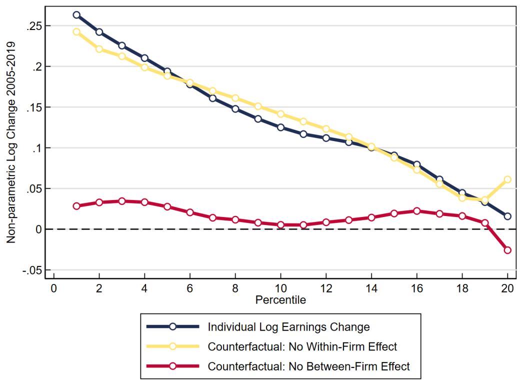
Figure A1: Evolution of Portuguese Wage Inequality (2005-19).



(a) Inequality dynamics



(b) Between firm inequality decomposition



(c) Nonparametric density decomposition

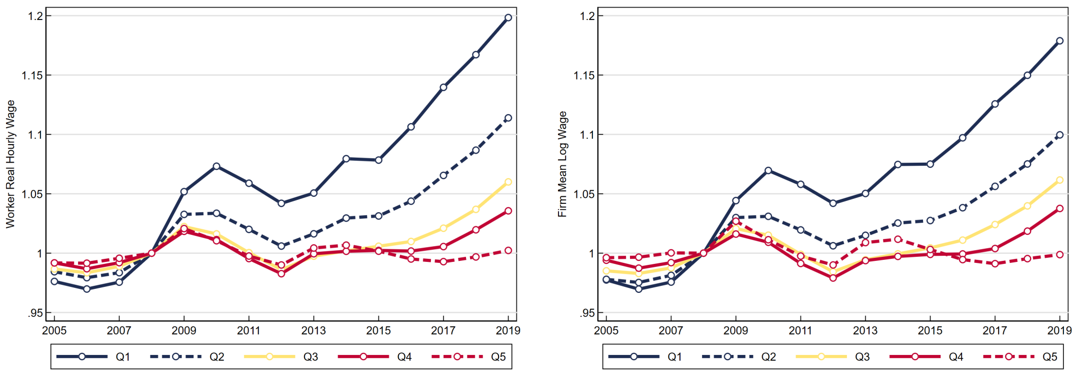
Source: *Quadros de Pessoal*, $2005-19$.
Note: Panel (a) plots three inequality measures, the Gini coefficient of real hourly wage, the variance of log real hourly wages, and the log ratio of real hourly wage at the 90^{th} and 10^{th} income percentiles, between 2005 and 2019. Real hourly wages are gross nominal wages of full-time dependent workers deflated by the consumer price index (base=2015). The 1^{st} and 99^{th} percentiles of real hourly wages were trimmed every year. Panel (b) is built using the procedure described in Barth et al. (2016), Alvarez et al. (2018), Song et al. (2019), and Juhn et al. (1993). It shows the increase in log earnings between 2004 and 2019 within each percentile of the earnings distribution (blue line). To build the red line, for a given percentile, we take firm mean earnings and average them across all the employers of workers in that percentile separately in both years, and then take the difference over the period. The within line will be automatically the same as the difference between the other two lines. Panel (c) shows the results of the non-parametric density decomposition described in Machado and Mata (2005), Autor et al. (2005), and Song et al. (2019). To produce this figure, we first compute two sets of statistics each for 2005 and 2019. First, we obtain the percentiles of the distribution of firms' mean log hourly earnings, weighted by employment. Then, within each percentile, we calculate 500 quantiles of the distribution of the difference between log worker hourly earnings and the average earnings in that firm-based percentile. These two sets of bins are subsequently used to produce the counterfactual distributions shown in Panel (c). For additional detail on this procedure, please refer to Song et al. (2019)'s Online Appendix E.

Table A1: Mobility Matrix

	2005 - 09	2010 - 14	2015 - 19
Number of unique workers	2.25	2.06	2.15
Average number of firms	1.24	1.18	1.24
Average number of job titles	1.54	1.89	1.63
% Job title switchers	.43	.70	.49
% Firm switchers	.21	.17	.21

Source: *Quadros de Pessoal*, $2005-19$.
Note: The number of unique workers is in millions. A switcher is defined as a worker who is associated with two or more employers (or two or more job titles) during the period. For example, we interpret the value of 0.21 in the first column, last row, as follows: 21 percent of workers present in the 2005-09 sub-period switched employer at least once.

Figure A2: Change in Percentiles of Annual Earnings Overall and Between Firms

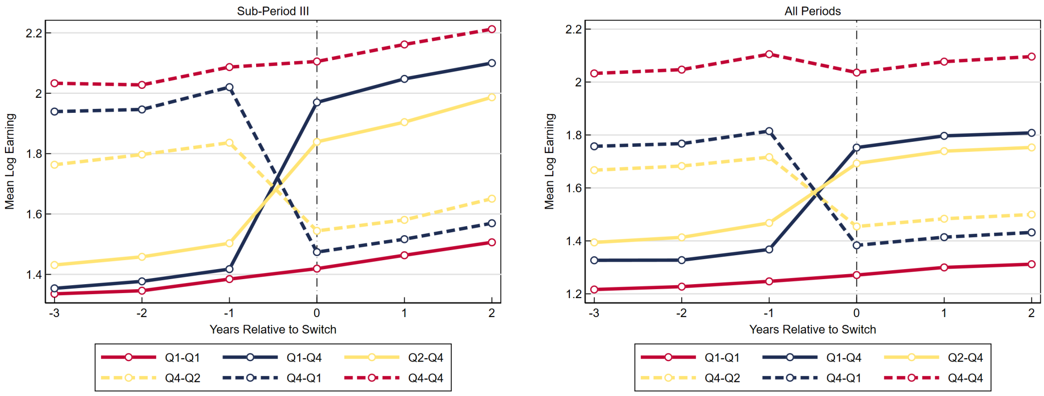


(a) Overall earnings

(b) Between firms

Source: *Quadros de Pessoal*, $2005-19$.
Note: Panel (a) plots the dynamics of log hourly earnings for workers in five quintiles. To construct this figure, we average log hourly earnings by wage bin and year, and plot this metric over time. We have normalized this average to 1 in 2008. The widening of the curves – with lower quintiles growing faster – suggests wage inequality is decreasing as years go by. Panel B repeats this procedure but using a worker's firm average log hourly wages. Panel (b) is built by first finding each firm's mean log wage in each year. Then, we proceed to average this value within each year and earnings bin (weighted by employment). We have normalized this average to 1 in 2008. The construction of the metrics behind this figure closely mirrors that of Panel (b) Figure A1. The widening of these curves over time suggests inequality in average firm pay is decreasing over time. Moreover, the fact that the patterns observed in Panel (a) track those observed in Panel (b) suggests that the evolution of average firm pay drove the reduction in inequality.

Figure A3: Change in Worker Earnings from Moving across Firm Quartiles.

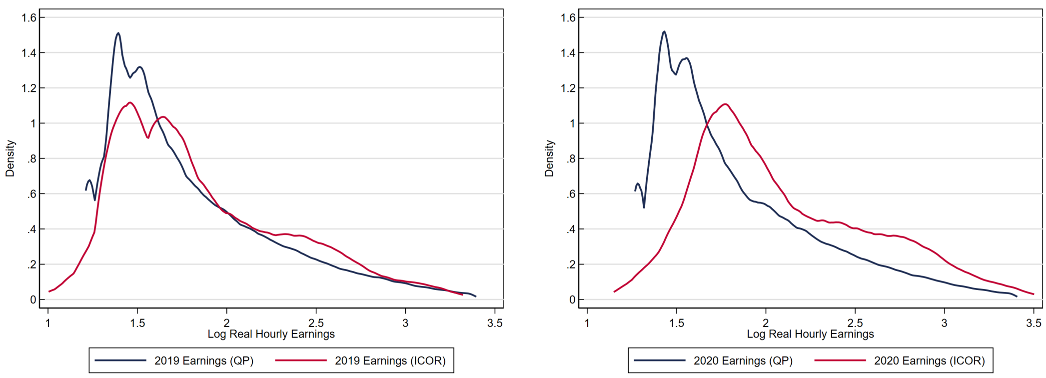


(a) Third subperiod

(b) All periods

Source: *Quadros de Pessoal*, $2005-19$.
Note: This figure presents for the third and all subperiods the average evolution in earnings of workers moving across quartiles of firm estimated AKM fixed-effects. In the main text (in figure 4), we have presented the same results for sub-periods one and two. To do these computations, we consider only workers who changed their jobs at most once during each subperiod. The solid blue line, for instance, presents the year-on-year average of log hourly wages of workers having moved from the lowest quartile to the highest quartile, where years are normalized in reference to the event. In this exercise, we require that workers are present at least two years prior and two years posterior to the job switch.

Figure A4: Earnings Distributions Comparison between QP and ICOR.



(a) 2019

(b) 2020

Sources: *Quadros de Pessoal*, $2019, 2020$; *EU-SILC*, $2019, 2020$.
Note: This figure provides evidence supporting the quality of the data. These figures present Kernel density comparisons for the wage distributions of 2019 and 2020, in *Quadros de Pessoal* and *Inquérito às Condições de Vida e Rendimento*. These Figures were built using the log of real hourly wages (in gross terms) of full-time dependent workers between ages 18 and 65. We use the consumer price index to convert both series to real terms. Observations from ICOR are weighted by means of cross-sectional sample weights provided by Statistics Portugal. In both years, and in both data sets, we have trimmed the 1^{st} and 99^{th} percentiles of real hourly wages.

Figure A5: AKM Residuals by Firm, Worker, and Job Title Fixed Effect Deciles (Second Period)

(a) Firm FE *vs.* worker FE (b) Job title FE *vs.* worker FE

Source: *Quadros de Pessoal*, $2005-19$.
Note: Panels (a) and (b) show the mean residuals per firm and worker fixed-effect deciles (panel (a)), but also per job and worker fixed effects (panel (b)) for the second subperiod. All fixed effect estimates come from the AKM model 1 estimated on the largest connected set. Figure 3 in the main text presents the results for the first subperiod.

Figure A6: AKM Residuals by Firm, Worker, and Job Title Fixed Effect Deciles (Third Period).

(a) Firm FE *vs.* worker FE (b) Job title FE *vs.* worker FE

Source: *Quadros de Pessoal*, $2005-19$.
Note: Panels (a) and (b) show the mean residuals per firm and worker fixed-effect deciles (panel (a)), but also per job and worker fixed effects (panel (b)) for the third subperiod. All the fixed effect estimates come from the AKM model 1 estimated on the largest connected set. Figure 3 in the main text presents the results for the first subperiod.

Table A2: Robustness Checks of the Variance Decomposition

	2005		2019		Δ	
	Total Var	Between-firm	Total Var	Between-firm	Δ Total Var	Δ Between (%)
All	0.328	0.214	0.255	0.149	-0.073	88.677
Demean: Region	0.301	0.187	0.243	0.137	-0.058	85.911
Demean: Broad industry	0.255	0.140	0.213	0.107	-0.042	80.336
Demean: 2-digit industry	0.240	0.125	0.190	0.084	-0.050	83.567
Demean: Gender	0.315	0.204	0.247	0.141	-0.069	91.399
Demean: Birth cohort	0.312	0.202	0.247	0.145	-0.064	88.491
Demean: Nationality	0.327	0.212	0.254	0.148	-0.073	88.904
Demean: Education	0.252	0.144	0.205	0.106	-0.047	80.645

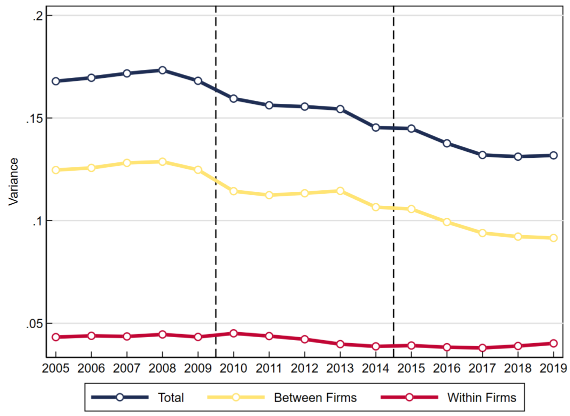
Source: *Quadros de Pessoal*, $2005-19$.
Note: This table provides robustness checks for the within-between firm variance decomposition. Total Var stands for total variance of log hourly real wages in a given year, while Between-firm stands for variance in average firm pay in a given year (weighted by employment). Δ Total Var denotes the absolute value change in total variance, while the last column presents the fraction of this change accounted for by changes in between-firm variance. Except for the first row, all statistics are computed using earnings demeaned within a given group, *before* all variances are calculated. This table shows that even within narrowly defined sectors or demographic groups, most of the decline in earnings inequality occurred between firms.

Table A3: Summary Statistics for Average Firm Regressors by Subperiods

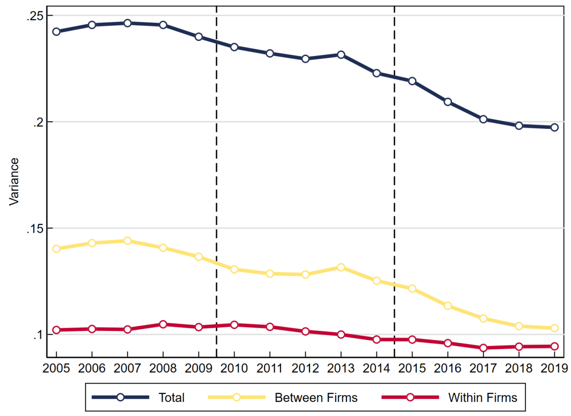
	2005 - 2009		2010 - 2014		2015 - 2019	
	Mean	SD	Mean	SD	Mean	SD
Av. product market HHI	0.022	0.053	0.026	0.056	0.027	0.055
Av. labor market HHI	0.046	0.099	0.053	0.108	0.056	0.110
Av. log of labor productivity	10.109	0.783	9.960	0.806	10.105	0.757
Av. firm skill requirement	0.369	0.115	0.395	0.118	0.393	0.118
Av. firm size	7.104	56.535	7.799	67.209	8.401	73.952
Av. share of min. wage workers	0.341	0.393	0.347	0.391	0.395	0.399

Source: *Quadros de Pessoal*, $2005-19$.
Note: This table presents descriptive statistics for the chosen firm covariates. These statistics are based on the mean firm covariates across years within subperiods. Firm size is based on the number of full-time employees in the firm, in October. Labor productivity is defined as value added per worker.

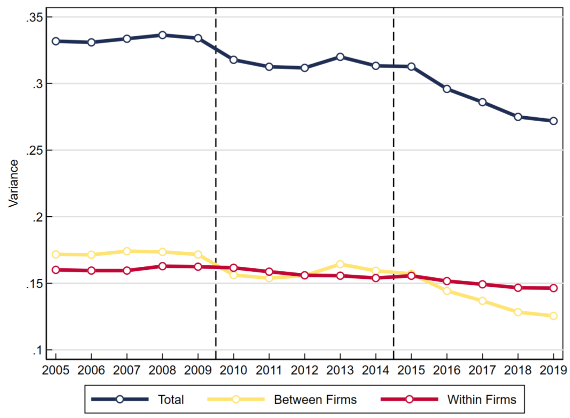
Figure A7: Within- and Between-Firm Inequality, by Firm Size (2005-19).



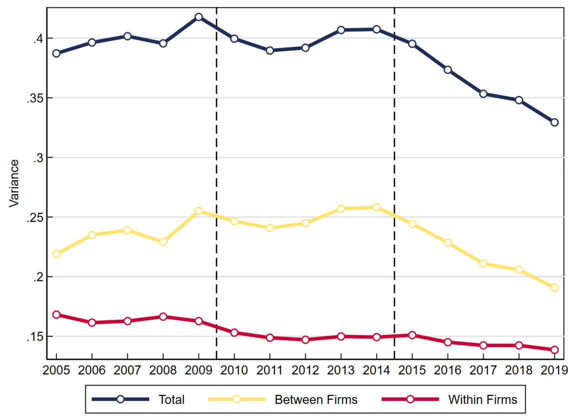
(a) 1st quartile of firm size



(b) 2nd quartile of firm size



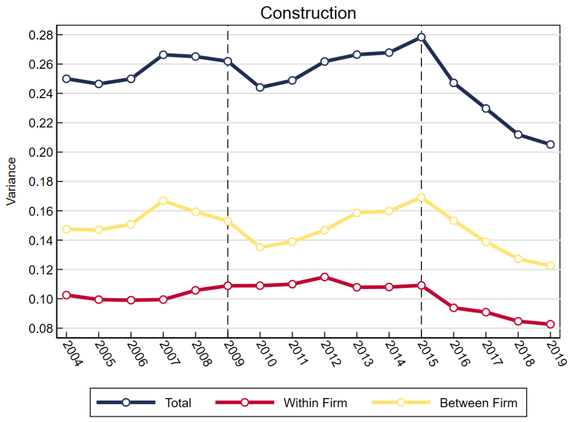
(c) 3rd quartile of firm size



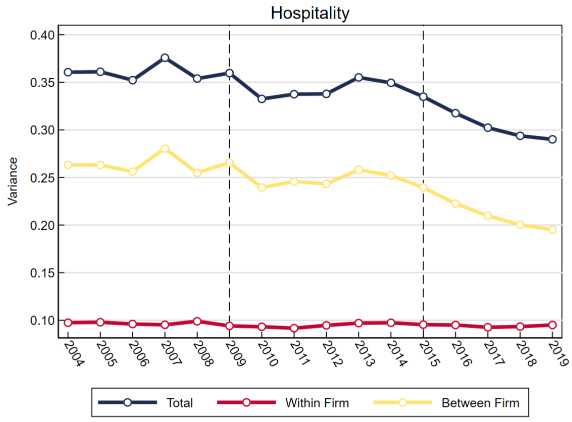
(d) 4th quartile of firm size

Source: *Quadros de Pessoal*, $2005 - 19$.
Note: Panels (a) to (d) plot the yearly evolution of the variance of hourly wages ("total wage inequality") over 2005-19, decomposed into a within-firm inequality and a between-firm inequality components by quartiles of firm size. Quartiles of firm size are constructed based on the average number of workers in the firm during the entire 2005-19 period. The vertical sum of the within- and between-firm inequality components adds up to overall inequality, for each year. Firm variance is computed based on average log earnings and is weighted by the number of workers in the firm. Within-firm variance is based on the difference between a worker's log hourly earnings and the average wage paid by his or her firm. Additional details on how to implement this estimation are provided in Appendix B.

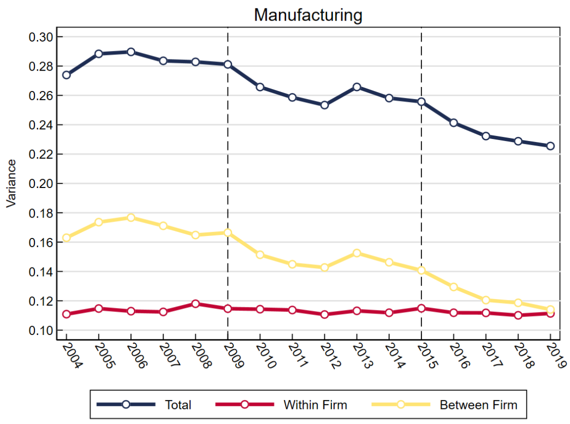
Figure A8: Within- and Between-Firm Inequality, by Sector (2005-19).



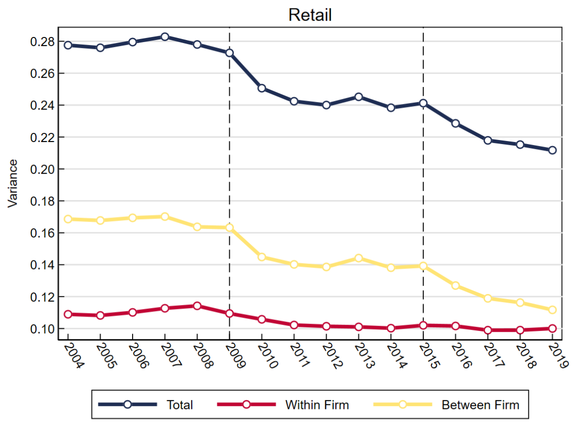
(a) Construction sector



(b) Hospitality sector



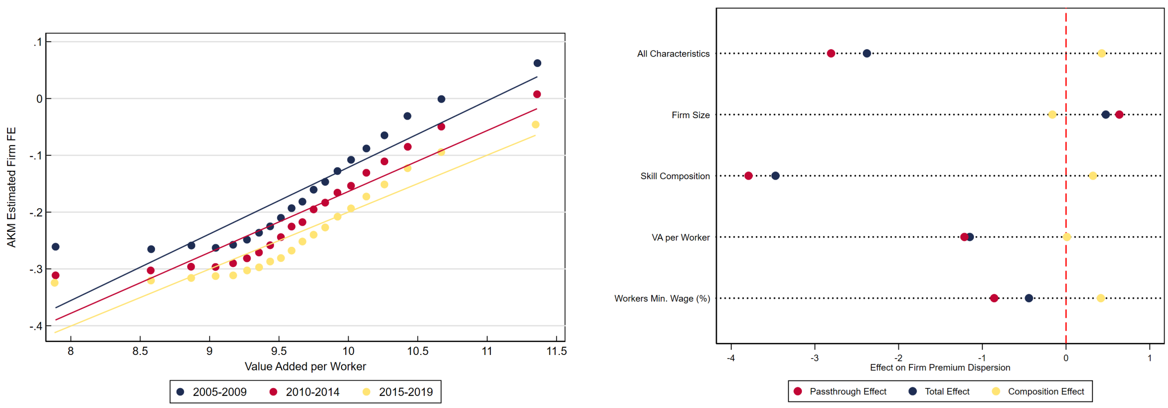
(c) Manufacturing sector



(d) Retail sector

Source: *Quadros de Pessoal*, $2005-19$.
Note: Panels (a) to (d) plot the yearly evolution of the variance of hourly wages ("total wage inequality") over 2005-19, decomposed into within-firm inequality and between-firm inequality components for selected sectors: construction, hospitality, manufacturing and retail. The vertical sum of the within- and between-firm inequality components adds up to overall inequality, for each year. Firm variance is computed based on average log earnings and is weighted by the number of workers in the firm. Within-firm variance is based on the difference between a worker's log hourly earnings and the average wage paid by his or her firm. Additional details on how to implement this estimation are provided in Appendix B.

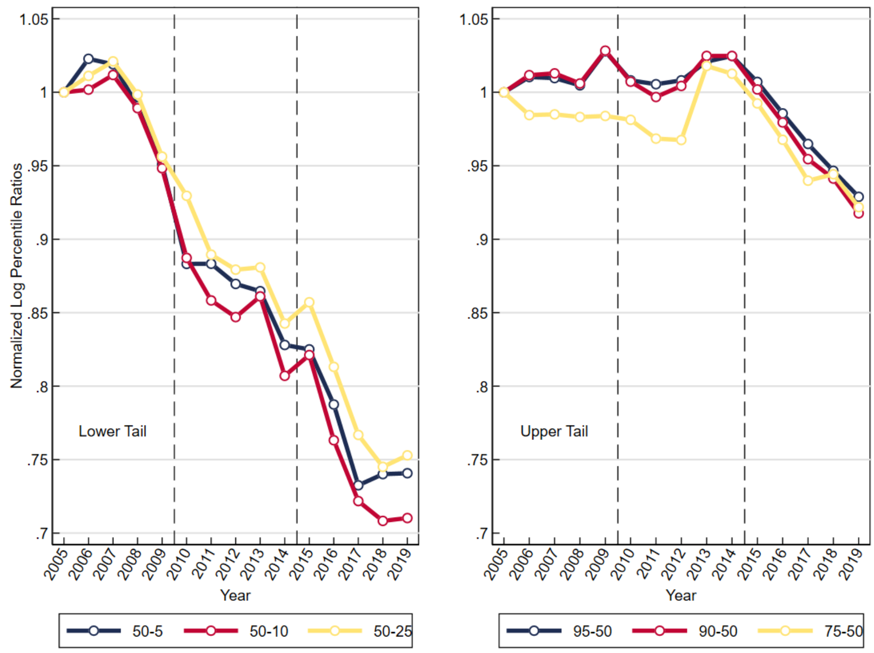
Figure A9: Declining Returns to Firm Characteristics and Composition



(a) Firm effects *vs.* value added per worker (b) Passthrough *vs.* composition

Source: *Quadros de Pessoal*, 2005 – 19.
Note: Panel (a) shows the average estimated firm effect in each subperiod against value added per worker (by 20 bins of log mean value added per worker in the subperiod). Value added has been constructed by averaging value added per worker at the firm level over each subperiod, and then taking logs. Overlaid are ordinary least squares best fit lines, whose slope capture returns to value added. Panel (b) presents the key messages from Table 6 in a graphical manner. Blue dots represent the contribution of each characteristic to the decline in firm pay premium dispersion. Red dots show the portion of this contribution due to passthrough effects and yellow dots show the portion due to composition effects. The horizontal sum of the yellow and red dots must add up to the blue dots by construction.

Figure A10: Wage Inequality Dynamics in Portugal: Upper and Lower Tails (2005-19)



Source: *Quadros de Pessoal*, 2005 – 19.
Note: The figure plots different measures of inequality for the lower and upper tails of the distribution for 2005-19. The inequality measures are normalized to 1 for 2005, and present the evolution of the indicators over time. This figure shows that inequality decreased across the entire earnings distribution but was more pronounced at the bottom of the earnings distribution.

Table A4: Projection of Covariates into Firm Pay Premium Variance (All Periods)

	RIF$_{(\sigma^2_{\hat{\psi}})}$ - Firm fixed effects variance			
	2005-09	2010-14	2015-19	2005-19
Firms size (log)	-0.007***	-0.008***	-0.006***	-0.006***
Value added per worker (log)	0.003***	0.003***	0.002***	0.002***
Product concentration	0.044***	0.015***	0.014***	0.042***
Labor market concentration	0.007***	-0.001	-0.000	0.004***
Share of minimum wage workers	0.083***	0.046***	0.038***	0.069***
Firm skill requirement	0.180***	0.112***	0.079***	0.217***
N	6,779,289	6,301,025	6,595,153	22,192,961
R^2	0.051	0.081	0.067	0.110
Significance:				*p<0.1; **p<0.05; ***p<0.01

Source: *Quadros de Pessoal*, $2005-19$.

Note: The table displays the coefficients obtained when projecting the covariates into the estimated firm pay premium variance (RIF$_{(\sigma^2_{\hat{\psi}})}$). All the estimations presented in the table control for sector and region. The standard errors are calculated by bootstrap, using 500 repetitions.

Table A5: Projection of Covariates into Job Title Pay Premium Variance (All Periods)

	RIF$_{(\sigma^2_{\hat{\phi}})}$ - Job-title Fixed Effects variance			
	2005-2009	2010-2014	2015-2019	2005-2019
Firms size (log)	0.002***	0.002***	0.002***	0.002***
Value added per worker (log)	0.002***	0.001***	0.001***	0.002***
Product concentration	-0.002	0.039***	0.061***	0.022***
Labor market concentration	0.039***	0.011***	-0.009***	0.014***
Share of minimum wage workers	0.009***	0.006***	0.003***	0.008***
Firm skill requirement	0.066***	0.036***	0.057***	0.083***
N	6,779,289	6,301,025	6,595,153	22,192,961
R^2	0.181	0.190	0.075	0.202
Significance:				*p<0.1; **p<0.05; ***p<0.01

Source: *Quadros de Pessoal*, $2005-19$.

Note: The table displays the coefficients obtained when projecting the covariates into the estimated job title pay premium variance (RIF$_{(\sigma^2_{\hat{\phi}})}$). All the estimations presented in the table control for sector and region. The standard errors are calculated by bootstrap, using 500 repetitions.

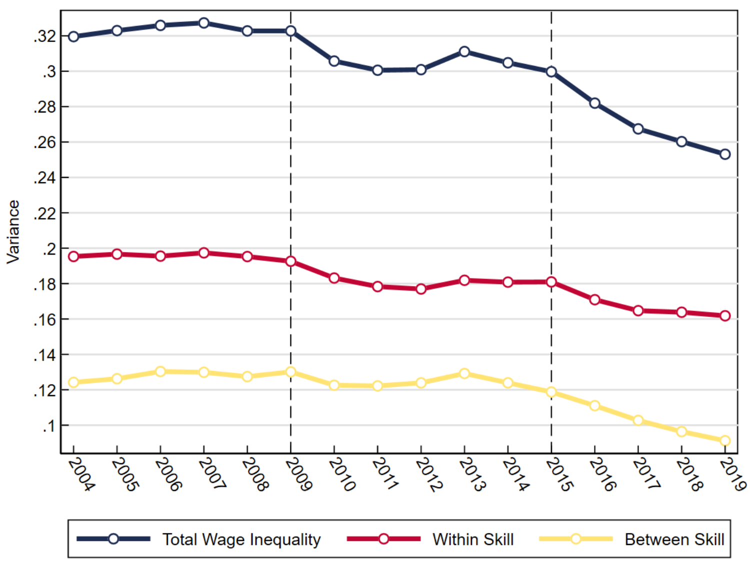
Table A6: Projection of Covariates into Firm-Job Title Pay Premium Covariance (All Periods)

	RIF$_{(\sigma^2_{\hat{\psi},\hat{\phi}})}$ - Firm-Job title fixed effects covariance			
	2005-09	2010-14	2015-19	2005-19
Firms size (log)	0.001***	0.001***	0.001***	0.001***
Value added per worker (log)	0.001***	0.001***	0.001***	0.002***
Product concentration	-0.006***	0.010***	0.014***	0.014***
Labor market concentration	0.015***	0.005***	-0.003***	-0.000*
Share of minimum wage workers	0.018***	0.009***	0.007***	0.013***
Firm skill requirement	0.032***	0.019***	0.022***	0.034***
N	6,779,289	6,301,025	6,595,153	22,192,961
R^2	0.157	0.204	0.128	0.176

Significance: *p<0.1; **p<0.05; ***p<0.01

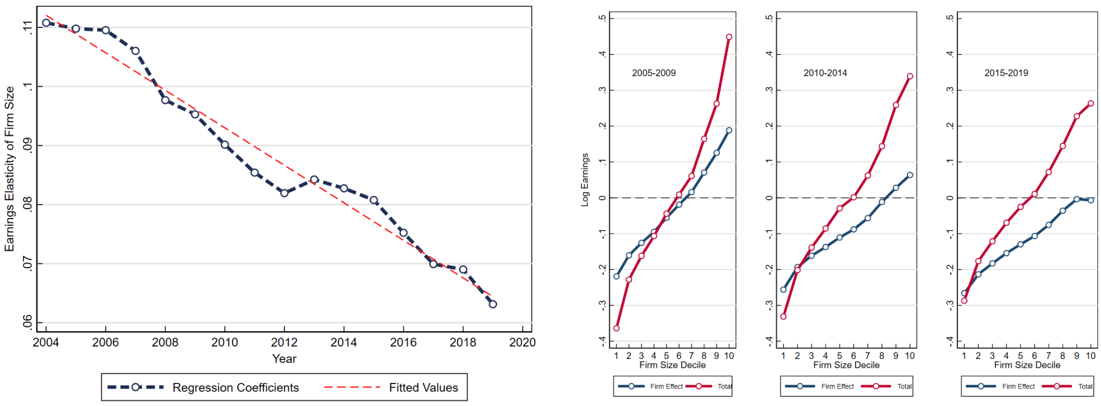
Source: *Quadros de Pessoal*, 2005 – 19.
Note: The table displays the coefficients obtained when projecting the covariates into the estimated firm-job title pay premium covariance (RIF$_{(\sigma^2_{\hat{\psi},\hat{\phi}})}$). All the estimations presented in the table control for sector and region. The standard errors are calculated by bootstrap, using 500 repetitions.

Figure A11: Between- and Within-Skill Group Inequality (2005-19)



Source: *Quadros de Pessoal*, 2005 – 19.
Note: The figure shows the inequality decomposition of labor income inequality between and within skill groups. To build this figure, we started by running a Mincer-type regression of log hourly wages on education, tenure, gender, and all possible interactions between these variables. Taking the variance of each side of this estimated model yields the between- and within-skill components of inequality (the variance of the predicted component being between skill inequality, while the variance of the predicted residual can be seen as within skill inequality).

Figure A12: Wages and Firm Size in Portugal (2005-19)



(a) Declining wage-size elasticity

(b) Declining firm premium to large firm

Source: *Quadros de Pessoal*, $2005-19$.
Note: Panel (a) plots the coefficient that results from projecting labor earnings onto the size of the firm (by year) using $\log(w_{ij}) = \alpha_o + \alpha_1 \log(N_j) + \epsilon_{ij}$. Panel (b) plots the average firm effects and average log earnings per firm size decile. Firms are assigned to 10 size classes. Following Bloom et al. (2018), we plot the average log earnings in each firm size class relative to total average log earnings over the interval and firm fixed effects components estimated using the AKM equation. We omit worker fixed effects and the residual component for the sake of readability. Each panel displays these results for a different five-year interval. The fact that the blue schedule is flattening over time (say, going from the first panel to the second) suggests that moving from a large to a very large firm is being less rewarded over time. However, at the bottom of the firm size distribution, moving from a small to a medium-sized firm still yields a substantial premium.

Table A7: Mediation Analysis of the Fixed Effects Components in Wage Level

	Restricted model	Full model	Difference	Mediation component		
	β_0	β_1	$(\beta_0 - \beta_1)$	Worker FE	Firm FE	Job-title FE
	(1)	(2)	(3)	(4)	(5)	(6)
Firms size (log)	0.032	0.037	-0.005	0.004	-0.011	0.002
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Value added per worker (log)	0.027	0.002	0.025	0.013	0.009	0.003
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Product concentration	0.448	-0.009	0.457	0.181	0.145	0.131
	(0.000)	(0.001)		(0.000)	(0.000)	(0.000)
Labor market concentration	-0.468	-0.033	-0.435	-0.215	-0.085	-0.135
	(0.008)	(0.008)		(0.006)	(0.004)	(0.004)
Share of minimum wage workers	-0.553	-0.283	-0.270	-0.121	-0.113	-0.037
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Firm skill requirement	1.360	-0.089	1.449	0.797	0.210	0.443
	(0.001)	(0.001)		(0.000)	(0.000)	(0.000)

Source: *Quadros de Pessoal*, $2005-19$.

Table A8: Mediation Analysis of the Fixed Effects Components in Wage Variance

	Restricted model β_0 (1)	Full model β_1 (2)	Difference $(\beta_0 - \beta_1)$ (3)	Mediation component Worker FE (4)	Firm FE (5)	Job-title FE (6)
Firms size (log)	0.021	0.012	0.010	0.012	-0.004	0.002
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Value added per worker (log)	0.019	-0.000	0.019	0.012	0.005	0.003
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Product concentration	0.476	-0.089	0.566	0.200	0.193	0.173
	(0.001)	(0.002)		(0.000)	(0.000)	(0.000)
Labor market concentration	-0.632	0.018	-0.650	-0.295	-0.189	-0.166
	(0.011)	(0.013)		(0.008)	(0.004)	(0.004)
Share of minimum wage workers	0.223	0.130	0.094	0.019	0.039	0.035
	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
Firm skill requirement	1.179	-0.011	1.190	0.743	0.241	0.205
	(0.001)	(0.002)		(0.000)	(0.000)	(0.000)

Source: *Quadros de Pessoal*, $2005-19$.

Table A9: Summary Statistics and Variance Decomposition - Longer Periods (i)

		2005 - 11		2012 - 18	
Largest connected set					
	Observations	11706464		10892203	
	Firms	338494		267601	
	Workers	2633341		2352107	
	Job titles	38717		48347	
	Movers across firms	622956		540507	
	Movers across job titles	1284898		1310323	
	Mean $\log(w)$	1.7161		1.7531	
Variance components					
	Variance $\log(w)$	0.3209		0.2923	
	Variance workers fix effects	0.1307	40.73	0.1287	44.02
	Variance firms fix effects	0.0696	21.69	0.0541	18.52
	Variance job-title fix effects	0.0168	5.24	0.0157	5.36
	Coefficient of determination - R^2	0.9057		0.9159	

Source: *Quadros de Pessoal*, $2005-18$.
Note: The table displays summary statistics and the labor income variance decomposition (for a set of selected components of interest) for two different subperiods. The subperiods in this table (7 years) are longer than the subperiods presented in the main body of the paper (5 years). The subperiods start in 2005. The variance decomposition follows equation 4 in the text on the restricted AKM especification (equation 1). The decomposition is performed in the largest connected set.

Table A10: Summary Statistics and Variance Decomposition - Longer Periods (ii)

		2006 - 2012		2013 - 2019	
Largest connected set					
	Observations	11583790		11058898	
	Firms	330077		266424	
	Workers	2597959		2417569	
	Job titles	51091		47167	
	Movers across firms	587026		575254	
	Movers across job-titles	1641026		1296282	
	Mean log(w)	1.7202		1.7696	
Variance components					
	Variance log(w)	0.3181		0.2857	
	Variance workers fix effects	0.1366	42.95	0.1259	44.07
	Variance firms fix effects	0.0663	20.84	0.0512	17.92
	Variance job title fix effects	0.0152	4.77	0.0149	5.23
	Coefficient of determination - R^2	0.9074		0.9126	

Source: *Quadros de Pessoal*, $2006-19$.
Note: The table displays summary statistics and the labor income variance decomposition (for a set of selected components of interest) for two different subperiods. The subperiods in this table (7 years) are longer than the subperiods presented in the main body of the paper (5 years). The subperiods start in 2006. The variance decomposition follows equation 4 in the text on the restricted AKM especification (equation 1). The decomposition is performed in the largest connected set.

Table A11: Summary Statistics and Variance Decomposition - Overlapping

		2005 - 2010		2009 - 2015		2014 - 2019	
Largest connected set							
	Observations	9904497		10979592		9488457	
	Firms	316083		292998		249880	
	Workers	2439561		2394213		2294268	
	Job titles	36918		57717		45437	
	Movers across firms	6550		3784		3367	
	Movers across job-titles	14846		10286		9794	
	Mean log(w)	1.7153		1.7417		1.7760	
Variance components							
	Variance log(w)	0.3247		0.3094		0.2812	
	Variance workers fix effects	0.1364	42.02	0.1389	44.90	0.1289	45.84
	Variance firms fix effects	0.0739	22.75	0.0610	19.72	0.0507	18.04
	Variance job title fix effects	0.0162	4.98	0.0159	5.15	0.0142	5.05
	Coefficient of determination - R^2	0.9086		0.9152		0.9136	

Source: *Quadros de Pessoal*, $2005-19$.
Note: The table displays summary statistics and the labor income variance decomposition (for a set of selected components of interest) for three different subperiods that overlap. The variance decomposition follows equation 4 in the text on the restricted AKM specification (equation 1). The decomposition is performed in the largest connected set.

B Stylized Facts on Earnings Inequality in Portugal

In this section, we present the first set of stylized facts on Portugal's rapid decrease in earnings inequality between 2005 and 2019. Wage inequality in Portugal declined continuously over the course of the twenty-first century, by a staggering 20 percent. It would be difficult to determine a priori the direction and effect of firm and institutional characteristics in the evolution of wage inequality in Portugal. Instead, we limit ourselves to reporting some stylized facts that guide our analysis.

***i)* Heterogeneity in the Change in Inequality along the Wage Distribution** Although overall inequality has decreased in Portugal over the course of the twenty-first century, various demographic groups along the distribution may have been impacted differently. In what follows, we analyze what happened to (i) the lower tail of the distribution, (ii) the upper tail, and (iii) the distribution as a whole.

Figure A10, in Appendix A, presents measures of inequality over 2005-19. The figure shows that the decrease in inequality was driven by the lower tail of the distribution: looking at the normalized log percentile ratios, we see that convergence toward the median of the income distribution occurred at a faster pace for the percentiles below the median, compared to those above the median (corroborating evidence is provided in Figure A2). The fact that we also observe a decrease in inequality in the upper tail of the distribution suggests that the decline in inequality happened along the full support of the income distribution. As a formal assessment of whether inequality *unambiguously* went up or down over the considered period, we evaluate the Lorenz criterion for the log of real hourly wages in Portugal. Specifically, we say that given two distributions, X^{2005} and X^{2019}, X^{2019} Lorenz dominates X^{2005} if and only if

$$L_{X^{2019}}(p) \geq L_{X^{2005}}(p) \; \forall p \text{ with } > \text{ for some } p \tag{A1}$$

If this holds, and if the Lorenz curves do not cross (since this assures the completeness of the criterion), we can state that X^{2019} is *unambiguously* less unequal than X^{2005}. To perform the exercise empirically, we leverage Gastwirth (1971)'s identity to estimate

$$L_{X^{2019}}(p) - L_{X^{2005}}(p) \Leftrightarrow \frac{1}{\mu^{2019}} \int_o^p Q_X^{2019}(t)dt - \frac{1}{\mu^{2005}} \int_o^p Q_X^{2005}(t)dt \tag{A2}$$

The next step is to evaluate whether this differential is positive or negative for $\forall p$. In the expression above, $Q_X(t)$ is the quantile function for the given distribution ("Pen's Parade", the inverse of the cumulative distribution function), so that estimating $\int_o^p Q_X(t)dt$ boils down to estimating the generalized Lorenz curve. When scaled down by the mean of the distribution, μ, the Generalized Lorenz curve becomes the Lorenz curve. The application of this criterion to Portuguese data for 2005 and 2019 reveals that the decrease in inequality was unambiguous and took place along the entire wage distribution. We show the application of this criterion in the Online Appendix. We compare the Lorenz curves of the distributions at the beginning and end of the period considered (Atkinson, 2008; Gastwirth, 1971). This exercise supports the claim that inequality *unambiguously* decreased in Portugal along the support of the distribution. The Lorenz curve for 2019 stochastically dominates the Lorenz curve for 2005, and there are no intersections.

***ii)* Earnings Dispersion between and within Firms** Next, we decompose wage inequality into the contributions of within- and between-firm inequality. This provides some preliminary understanding on the role of firm heterogeneity. If all firms paid the same wage to all employees, there would be no within-firm inequality, but not necessarily no wage inequality as firms could still differ in the wages that they pay. Likewise, if all firms had the same distribution of wages, there would be no inequality between-firms, but not necessarily no wage inequality as workers within each firm could earn different wages. These are the two extreme cases. With this in mind, we examine which of these factors was more prominent in Portugal between 2005 and 2019, shedding light on whether wage dispersion was mostly driven by systematic differences in pay premiums across firms or differences in pay within each firm. To do so, we decompose the variance of wages into its between and within components. Following Alvarez et al. (2018), Song et al. (2019), and Messina and Silva (2021) wages can be decomposed by construction as:

$$w_t^{i,j,f} \equiv \overline{w_t} + (\overline{w_t^f} - \overline{w_t}) + (w_t^{i,j,f} - \overline{w_t^f}) \tag{A3}$$

where $w_t^{i,j,f}$ is the log of real hourly wages of worker i in firm f in year t, $\overline{w_t}$ is the average log of the real hourly wage in the economy in year t, and $\overline{w_t^f}$ is the average log of the real hourly wage in firm f (where worker i works) in year t. The wage of each worker can be seen as the sum of the average remuneration in the economy in that year, the difference paid on average by firms relative to the average wage in the economy, and the difference earned by workers relative to their firm's average wage. To obtain the within- and between-firms components of wage variance in each year, we rearrange and transform this identity into:

$$Var(w_t^{i,j,f} - \overline{w_t}) = Var(\overline{w_t^f} - \overline{w_t}) + Var(w_t^{i,j,f} - \overline{w_t^f}) \tag{A4}$$

where $Cov(\overline{w_t^f} - \overline{w_t}; w_t^{i,j,f} - \overline{w_t^f}) = 0$ by construction.[29] Since wage variance is decomposed yearly, equation A4 becomes

$$Var(w_t^{i,j,f}) = Var(\overline{w_t^f}) + \sum_{f=1}^{N} \omega_f Var(w_t^{i,j,f} | i \in f) \Leftrightarrow \tag{A5}$$

$$\underbrace{Var(w_t^{i,j,f})}_{Overall\ Inequality} = \underbrace{Var(\overline{w_t^f})}_{Between\ Firm\ Inequality} + \underbrace{\overline{Var(w_t^{i,j,f} | i \in f)}}_{Within\ Firm\ Inequality} \tag{A6}$$

This equation decomposes the yearly overall variance of log real hourly wages into the between-firm component (given by the variance across firm average wages), and a within-firm component (given by the weighted average of within-firm wage variance, with weight ω_f being the share of employment in firm f). Throughout the period, between-firm inequality accounted for over 60 percent of total wage inequality, and within-firm inequality accounted for slightly less than 40 percent (see Figure 1). In the subperiods considered (2005-09, 2010-14, and 2015-19) within- and between-firm inequality moved broadly in the same direction driving the overall change in inequality. However, the stronger reduction of inequality in 2010-14 and 2015-19 was mostly driven by the reduction in inequality between-firms. To verify that the observed patterns of between-

[29] $Cov(\overline{w_t^f} - \overline{w_t}; w_t^{i,j,f} - \overline{w_t^f}) = E([\overline{w_t^f} - \overline{w_t} - E(\overline{w_t^f} - \overline{w_t})][w_t^{i,j,f} - \overline{w_t^f} - E(w_t^{i,j,f} - \overline{w_t^f})]) = 0$

and within-firm inequality are not driven by specific sectors but are representative of the economy as a whole, we further run this equation for four selected sectors: manufacturing, construction, retail, and hospitality (see Figure A8, in Appendix A). Our key insight holds regardless of the broad sector being considered. The same holds if we repeat the decomposition by firm size (see Figure A7).

***iii)* Earnings Dispersion between and within Skills** The richness of our data also allows us to calculate inequality between different skill groups and assess how this measure has changed over time. This exercise reveals the prominence of systematic differences in returns to skills across different skill types in determining wage dispersion. To disentangle overall wage inequality, we follow Messina and Silva (2021). We start by running a standard Mincerian equation of the form $w_{it} = \rho_t \mathbf{X}_i + \mu_{it}$, where w_{it} stands for the log hourly wage of worker i in period t. $\mathbf{X}_i$ is a vector of covariates including a categorical educational level variable, tenure by five-year bins, and gender (as well as all possible interactions between these). ρ_t is a vector of returns to these covariates, and μ_{it} is an orthogonal error term, referred to as within-skill group wage inequality. Once estimated, we can apply variances to this relation to obtain

$$\underbrace{Var(w_{it})}_{Overall\ Inequality} = \underbrace{Var(\hat{\rho}_t \mathbf{X}_i)}_{Between-Group\ Skill\ Inequality} + \underbrace{Var(\hat{\mu}_{it})}_{Within-Group\ Skill\ Inequality} \tag{A7}$$

where we have used the orthogonality of the error term to impose zero covariance between the residual and the regressors. The variance of wages can thus be decomposed into a between-skill component and a within-skill component. Figure A11, in Appendix A, shows the results of implementing this decomposition. In levels, within-skill inequality accounts for the largest share of overall inequality (around 60 percent). In differences, however, between-skill inequality reduction seems to play a role that is roughly as important as within-skill inequality. Over 2005-19, around 50 percent of the reduction in wage inequality is attributed to the reduction in between-skill inequality, against 50 percent explained by within-firm inequality. If we zero in on the reduction in inequality witnessed over 2015-19, the reduction in between-skill inequality accounts for almost 60 percent of the overall reduction in inequality, despite its initially lower level. These findings highlight the importance of considering job title heterogeneity for wage dispersion.

***iv)* Decline in the Large Firm Pay Premium** The role of large firms as providers of better working conditions has been acknowledged in the past: in general, large firms offer better monetary and non monetary compensation. It is typical that in larger firms, jobs are more stable, there is greater worker satisfaction, and workers earn higher wages. However, there is evidence for the United States that the large-firm wage premium has been shrinking (Bloom et al., 2018). To assess whether this is the case in Portugal, we perform two exercises on the role of large firm size in the wage premium. First, we calculate the yearly elasticity of firm size with respect to wages.[30] Second, relying on the estimated firm effects from equation 1, we plot the (de-meaned) average log earnings and average fixed effects for each firm size decile, as in Bloom et al. (2018). This allows

[30]For each year between 2005 and 2019, we run the following specification: $\log(w_{ij}) = \alpha_o + \alpha_1 \log(N_j) + \epsilon_{ij}$.

us to assess the wage differential between different types of firms over time.

Panel (a) in Figure A12, in Appendix A, shows a declining relationship between firm size and wages. The wage-size elasticity plummeted from around 11 percent in 2004 to under 7 percent in 2019, which corresponds to a reduction of approximately 60 percent. Thus, the pay premium that large firms offer appears to have shrunk in absolute terms. This finding is backed by the results presented in panel (b) in Figure A12, where we explore the relationship between the firm pay premium and firm size along different sub-periods. The fact that the blue schedule flattens over time indicates that the returns to working in a large firm have declined over time. As large firms have historically paid significantly higher wages, it is important to understand the implications of a fall in the large firm wage premium for changes in inequality.